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Production-based asset pricing: a cross-industry study

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Production-based asset pricing: A cross-industry study

by

Zhi Wang

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1. INTRODUCTION

Many asset pricing models in the finance literature explain an asset's expected return by its covariance with other assets' returns. Although this approach may generate successful empirical results, it does not answer the question of what real risks cause an asset's expected return to vary. In the past two decades there has been a growing body of research relating financial asset returns with macroeconomic risks. Most of the theoretical and empirical studies in this area are carried out within two widely used frameworks: the consumption-based asset pricing model (CCAPM) and the production-based asset pricing model (PCAPM).

The consumption-based asset pricing model (CCAPM) is based on the intertemporal capital asset pricing model of Lucas (1978). The model assumes that there is a representative consumer in the economy and output evolves according to an exogenous Markov process. The representative consumer maximizes her additive and time-separable lifetime utility subject to a budget constraint. At each time period, the total endowment is allocated for current consumption and financial investment. The first-order condition characterizing the optimal consumption and investment decisions relates the asset return to the intertemporal marginal rate of substitution of consumption (IMRS). In addition, it can be shown that the risk premium of any financial asset is proportional to the negative of the covariance of the return with the IMRS. Within the CCAPM framework, the presence of macroeconomic risks can be inferred from their effects on consumption decisions.

Despite the appealing theoretical features, numerous empirical tests in the litera-

ture have not provided much supporting evidence for the CCAPM. Hansen and Single-
ton (1983) estimate and test a single good, representative consumer model with time-
additive and constant relative risk aversion (CRR) preferences. The asset returns used
to construct orthogonality conditions include the equally weighted average returns on
all stocks listed on the New York Stock Exchange (NYSE), the value-weighted aver-
age return on stocks on the NYSE, the equally weighted returns on the stocks of three
two-digit SEC industries, and the nominal returns on risk-free bonds. The empirical
results show that the orthogonality conditions are rejected at the 5 percent significance
level for almost all sets of returns. The authors conclude that the common stochas-
tic discount factor defined as the representative consumer's IMRS fails to capture the
relative risk structure of stocks versus bonds. Following Hansen and Singleton, more
extensive tests on CCAPM have been conducted by allowing for consumption durabil-
ity (Dunn and Singleton (1986)), choice of leisure (Mankiw, Rotemberg, and Summers
(1985) and Eichenbaum, Hansen, and Singleton (1988)), structural breaks (Ghysels and
Hall (1990)), habit persistence (Ferson and Constantinides (1991)), and nonexpected
utility preferences (Epstein and Zin (1991)), etc. However, the basic conclusions are
the same: The representative CCAPM can not explain the relative risk structure of
alternative asset such as the returns on stocks and bonds or the returns on bonds of
different maturities. The empirical failure of CCAPM is mostly due to the fact that
nondurable consumption growth barely moves over the business cycle, and that it is
poorly correlated with stock returns.

An alternative way to study the relationship between asset returns and macroeco-
nomic variables is the production-based asset pricing model (PCAPM), originally de-
veloped by Brock. Brock (1982) extends the asset pricing model of Lucas (1978) to
incorporate a nontrivial investment decision by modifying the stochastic growth model
of Brock and Mirman (1972). Thus, the general equilibrium PCAPM can be viewed as a
natural extension of the CCAPM by endogenizing the production process of an economy.

The PCAPM links asset returns to the marginal rate of transformation and infers the presence of macroeconomic risks from their effects on firms' investment decisions.

Compared with the empirical CCAPM literature, the empirical PCAPM literature is relatively small. Following Cochrane (1991), there has been a growing body of research testing empirical implications of the PCAPM. Why should we care about the empirical performance of PCAPM? Part of the reason is due to the fact that models focusing exclusively on the financial market and the CCAPM have not generated very satisfactory empirical results. Moreover, empirical study on PCAPM can shed lights on the following fundamental and important questions. How closely are the financial sector and the real sector related? Does physical capital investment convey crucial information on the pricing of financial assets? Is it necessary for economists to explicitly address the pricing impacts of key production characteristics in asset pricing models? In this study, I will address the above questions using industry-level data.

Before proceeding, I define two terminologies that will be used throughout the paper: *physical investment return* and *equity return*. The physical investment return is defined as the one-period gross rate of return on investing one dollar in physical capital (e.g., machine, office building, patent, etc.). The equity return is defined as the one-period gross rate of return on investing one dollar in financial security or equity portfolio.

Recent empirical studies on PCAPM include Cochrane (1991), Sharathchandra (1993), Bakshi, Chen and Naka (1995), Cochrane(1996), Kasa (1997), and Porter (1999). A detailed review on both the theoretical and the empirical literature about the PCAPM is provided in chapter 2. The existing empirical research on PCAPM has focused on using one or two aggregate production technologies and the corresponding physical investment return series to explain the risk structure of either aggregate or cross-sectional equity returns. For example, Cochrane (1996) considers two types of physical investment returns (nonresidential and residential investment returns) as factors to explain the variation in the expected returns of stock portfolios with different market capital-

ization. However, very few studies have been conducted in the literature linking industry physical investment returns to the corresponding industry equity portfolio returns. It is well known that different industries have different production characteristics and that equity returns vary across industries. Therefore, investigating the performance of PCAPM using cross-industry data is both intuitive and relevant. Specifically, I address the question of whether industry physical investment returns contain enough information to explain the variation in expected returns of the corresponding industry equity portfolios. Using industry-level physical investment returns has the following advantage over the traditional approach of relying on only one or two aggregate production technologies. The PCAPM implies that the variation in equity returns is driven by the effect of macroeconomic risks (e.g., productivity shocks) on the firm's physical capital investment. The magnitude of such effect is completely determined by the firm's production characteristics. Facing the same economy-wide shock, firms with different technologies will react quite differently. By allowing each industry to use a different technology, I essentially capture such heterogeneity and avoid losing useful pricing information due to inappropriate aggregation. Hence, the estimation and testing results are subject to fewer specification errors.

One dominant approach to empirically testing the PCAPM is to examine the validity of a factor pricing model for equity returns. Specifically, the only factors used to price equities are the physical investment returns. In other words, such a factor pricing model implies that the stochastic discount factor can be written as a linear combination of the physical investment returns. Cochrane (1996) initially adopts this approach and uses the generalized method of moments (GMM) to conduct parameter estimation and hypothesis test. The above approach suffers from two drawbacks. First, the linear factor pricing model may not be consistent with the spirit of no-arbitrage. The existence of the linear factor pricing model is guaranteed by the law of one price and an important assumption (the spanning assumption) stated below. However, the law of one price is much less

restrictive than the absence of arbitrage. Even if the law of one price is satisfied, there may still be arbitrage opportunities in the economy. Therefore, the constructed stochastic discount factor (a linear combination of physical investment returns) may take on negative values. This is not a desirable property for any asset pricing model. Second, estimating the factor pricing model in the GMM framework provides limited flexibility to expand the set of factors one wants to include in the stochastic discount factor. To avoid potential overparameterization problem in the GMM estimation, Cochrane selects only two aggregate production technologies to construct physical investment return series as factors. Hence, the above approach may suffer from a joint hypothesis test problem. Once the model is rejected, one is not clear whether rejection comes from the factor pricing model itself or from the inappropriate aggregation of production technologies. Further, the empirical results may also be sensitive to physical investment return series constructed from different aggregations of production technologies.

To construct a testing procedure inherently consistent with the spirit of no-arbitrage and to alleviate the joint hypothesis test problem by utilizing all of the relevant pricing information contained in industry-level physical investment return series, I propose an alternative method to investigate the pricing relationship between physical investment returns and equity returns using cross-industry data. Instead of testing the validity of the physical investment factor pricing model, I examine another closely related hypothesis, namely the *spanning assumption*. It states that *the payoff space of physical investment spans that of financial securities*. The relationship between the spanning assumption and the physical investment factor pricing model can be stated as follows. The law of one price implies that there always exists a discount factor that is a linear combination of the physical investment returns and the equity returns and that prices both. However, if one is willing to assume that financial securities offer no additional spanning opportunities on the payoff space beyond those offered by physical capital investment, as stated in the spanning assumption, then one can express the stochastic discount factor as a linear

combination of the physical investment returns only. Hence, the validity of the factor pricing model crucially depends on the validity of the spanning assumption. If the spanning assumption can not be rejected, then the data do not provide evidence against the physical investment factor pricing model. However, if the spanning assumption is rejected, then the factor pricing model will not hold, and one may have to look for new approaches to further studying the performance of the PCAPM or reject the PCAPM altogether.

Why should the payoff space of physical capital investment span the payoff space of financial securities? In general, physical investment and financial investment are just alternative ways of transforming goods across dates and states. If we assume that the financial securities traded on the New York Stock Exchange (NYSE) are claims to different combinations of M production technologies, then no-arbitrage constraints imply that the payoff space of physical capital investment should be exactly the same as the payoff space of financial securities in the absence of any market frictions (Cochrane (1991) and Porter (1999)). Hence, the most straightforward way to test the spanning assumption is to examine whether we can replicate the actual payoffs of financial securities by constructing the payoff space of physical capital investment. However, such approach is almost impossible to be implemented empirically due to the following two reasons. First, there is no way to guarantee that the selected production technologies and financial securities are able to span the entire payoff spaces of physical investment and financial investment. The sets of physical investment returns and equity returns included in the empirical study are most likely to span different parts of the payoff space. Second, even if we successfully replicate the entire payoff spaces of physical capital investment and financial investment, various market frictions (e.g., transaction costs) between the two markets may also easily break up the identity of the two payoff spaces. Therefore, instead of attempting to replicate the actual payoffs of financial investment by constructing the payoff space of physical capital investment, I will focus on exam-

ining whether one can infer all the relevant information necessary for pricing financial securities from physical investment returns. The above pricing relationship is an immediate implication of the spanning assumption and the PCAPM. The PCAPM implies that both the time-series and cross-sectional variations in equity returns are determined by the real macroeconomic risks through their effects on firms' physical capital investment. Hence, appropriately constructed physical investment returns should convey all the crucial information necessary for pricing the corresponding financial securities. On the other hand, the time-series and cross-sectional variations in equity returns should also reflect such pricing information.

To empirically study the pricing relationship between physical investment returns and equity returns, I propose a three-step procedure based on entropic principles and no-arbitrage constraints. The spanning assumption and no-arbitrage constraints jointly imply that any state price probability density (or risk-neutral probability measure) correctly pricing the physical investment returns should also be able to price the equity returns. Therefore, one natural way to test the validity of the spanning assumption is to recover the state price probability density from the physical investment return data, and examine whether it is consistent with the corresponding equity returns. As shown in chapter 4, the state price probability densities for both the physical investment market and the stock market can be recovered and compared by means of entropic principles. Focusing on the state price density instead of the stochastic discount factor allows me to avoid imposing parametric restrictions on the form of the stochastic discount factor. Hence, the nonparametric procedure using entropic principles is subject to fewer specification errors. Moreover, the estimated state price density derived from solving a minimum cross-entropy problem is by construction consistent with the absence of arbitrage. The data used in empirical testing includes physical investment return series and equity return series for each of the following six industries: mining, construction, manufacturing, transportation, communication, and public utilities. Empirical results

show that the state price density recovered from the physical investment returns can be used to correctly (in statistical sense) price the corresponding equity returns. Further examination shows that the above result is quite robust for a wide range of production parameters and different adjustment cost function forms. This provides supporting evidence that the spanning assumption holds at the cross-industry level.

One immediate implication of the above result is that physical capital investment contains crucial, if not exclusive, information about the effect of macroeconomic risks on financial asset pricing. Hence, any asset pricing model aiming to explain the cross-sectional variations in equity returns should at least capture such information. The model should either explicitly incorporate the presence of macroeconomic risks (e.g., productivity shock) affecting firms' physical capital investment or use appropriate proxies to capture such effect. This may provide an explanation for the empirical failure of the Capital Asset Pricing Model (CAPM) and the CCAPM. The traditional CAPM can be viewed as a linear factor pricing model with the market return as the only factor. In the context of PCAPM, market return may also be interpreted as a return generated by one aggregate production technology. Following the milestone papers of Markowitz (1952, 1959), Sharpe (1964), and Lintner (1965), numerous studies have emerged to test the empirical performance of the CAPM. Gibbons (1982) and Fama and French (1992) document the evidence that the traditional CAPM fails to explain the cross-sectional variations in stock returns. The empirical failure of the CAPM may be due to the fact that using one highly aggregated return (market return) as the only factor fails to capture some important intertemporal physical investment opportunities in the economy. In section 7.2, I present empirical evidence that the performance of linear factor pricing models are indeed sensitive to different levels of aggregation of the factors. Compared to the traditional CAPM, the empirical performance of the CCAPM is even more disappointing despite the fact that the model intends to capture the presence of macroeconomic risks through their effects on consumption decisions. In this case, con-

sumption change may just be a bad proxy for the effects of macroeconomic risks since nondurable consumption growth barely moves over the business cycle.

Another implication of the empirical findings in this study is that it is useful to develop asset pricing models that incorporate key characteristics in the production sector of the economy. For decades financial economists have focused most of their attention on the financial sector of the economy while developing asset pricing models. Relatively little effort has been made to explicitly model the pricing impacts of key production characteristics, e.g., the cost of adjusting capital stocks. Empirical studies (e.g., Malkiel, Furstenberg and Watson (1979)) have documented that adjustment cost has important impact on firm's physical capital investment decision. Since one central message from my study is that physical capital investment conveys crucial information on financial asset pricing, adjustment cost must have nontrivial impact on the prices of financial securities. The existence of adjustment cost implies that it is costly for a firm to adjust its capital stock. Hence, in response to a productivity shock, industries with higher adjustment cost adjust their capital stocks in a more sluggish manner than industries with lower adjustment cost. Since physical capital investment market and financial market are closely related, the above pattern exhibits itself in financial market with the following form: The equity prices for industries with higher adjustment cost exhibit more persistence than the equity prices for industries with lower adjustment cost. Basu (1987) makes a nice attempt to examine the impact of adjustment cost on the pricing, risk premia, and volatility of risky assets in an extended Brock's (1982) general equilibrium model. Clearly, models explicitly incorporating key production characteristics will have much richer implications on financial asset pricing than models exclusively focusing on the financial sector of the economy. More theoretical and empirical works need to be done to examine whether such models can capture both the time series and cross-sectional variations in expected equity returns.

In summary, my work contributes to the existing literature in three ways. First, I

conduct an extensive cross-industry study on the performance of PCAPM, which has not been thoroughly explored at the industry level. Specifically, I examine whether the payoff space of physical capital investment spans the payoff space of financial assets. Empirical results show that the state price density recovered from physical investment returns can be used to correctly price the corresponding equity returns. This provide supporting evidence for the spanning assumption and the physical investment factor pricing model. Second, instead of following the traditional approach of testing a linear physical investment factor pricing model, I propose an alternative procedure for testing the pricing relationship between physical investment returns and equity returns based on entropic principles. The proposed method is inherently consistent with the spirit of no-arbitrage while the traditional approach leads to a stochastic discount factor that may take negative values. Moreover, the new method provides more flexibility on efficiently extracting information on the production side of the economy than the traditional approach, and thus alleviates the joint hypothesis test problem and specification errors. Moreover, the proposed testing procedure is not restricted to test the implications of the PCAPM. It can be easily extended to study the pricing relationship between any two sets of asset returns. Third, the empirical results highlight the fact that physical capital investment conveys important information on financial asset pricing. Hence, to explain both the time-series and cross-sectional variations in equity returns, economists may have to explicitly model the impact of key production characteristics on asset prices.

The remaining of the paper is organized as follows. Chapter 2 provides a detailed review on the theoretical and empirical literature of the PCAPM. Chapter 3 derives the producer's first-order condition and a specific form of physical investment return. After introducing the physical investment factor pricing model and the spanning assumption in chapter 4, I formally presents the procedure for testing the spanning assumption within the entropic framework. Chapter 5 contains detailed data description and the construction of physical investment returns. Chapter 6 reports the empirical results and

robustness check. Chapter 7 tests the validity of the physical investment factor pricing model using the GMM approach and discusses how the state price density approach is related to the traditional approach. Chapter 8 concludes the dissertation with summary of results and discussion of future research.

2. LITERATURE REVIEW

In this chapter, I provide a detailed review on both the theory and the empirical tests of the PCAPM.

2.1 Theory of the PCAPM

The general equilibrium PCAPM should not be viewed as a substitute to the CCAPM. In fact they are complementary since the PCAPM is a natural extension of the CCAPM by endogenizing the production process of an economy. Brock (1982) extends the asset pricing model of Lucas (1978) to incorporate a nontrivial investment decision by modifying the stochastic growth model of Brock and Mirman (1972).

In Brock's model, the households own the initial capital stocks and competitive firms rent capital from households at a market-determined rental rate. Each firm issues one perfectly divisible equity share, representing claims to the firm's future profits. The representative household maximizes her intertemporal additive expected utility. In each period, she decides the amount of goods to consume, the amount of capital to invest, and the amount of equity shares to hold for the next period subject to her current budget constraint. There are a total of n firms in the economy, and each firm is allowed to have its own production technology. The objective of each firm is to maximize its current period profits by making optimal capital investments. At the end of each period, firms sell output, pay rents to the owners of the capital, pay dividends to shareholders, and return the undepreciated capital to their owners. The information structure of the

economy is as follows. All agents observe the realization of the economy-wide technology shock in each period. However, the households have to decide the allocation of capital stock across firms before the realization of technology shock and rental rates, while the firms decide how much capital to invest after observing the rental rates. Finally, for the above asset pricing model with production and capital accumulation, Brock defines a recursive competitive equilibrium with rational expectations.

The household's problem can be solved by stochastic dynamic programming techniques. The first-order conditions relate both the rate of return from lending capital goods and the rate of return from purchasing equity shares to the IMRS. The intuition behind this is obvious. Since physical capital investment and financial investment are just alternative ways to transform goods across dates and states, they should be priced by the same stochastic discount factor if no arbitrage opportunities are allowed. In this model, the stochastic discount factor is just the IMRS. Furthermore, using the results from the one-sector optimal growth model and applying a fixed-point contraction theorem, the equilibrium rental rates, the equilibrium dividends, and the equilibrium equity price can be solved as time-invariant functions of the economy-wide state variables: the pre-determined capital stock and the realized technology shocks. One immediate implication from this result is that the variations in equity returns are driven by the effect of macroeconomic risks on the firms' physical capital investment. Thus, information contained in physical capital investment may be valuable to capture both the time series and cross-sectional variations in equity returns.

2.2 Empirical Tests of the PCAPM

To empirically test a PCAPM, one can adopt either a general equilibrium approach or a partial equilibrium approach. The general equilibrium approach requires one to specify both the consumer side and the producer side of the economy. Although this

approach is consistent with the Brock's (1982) general equilibrium model, it requires imposing restrictions on preference assumptions in order to derive testable formulas. Since the general equilibrium model includes the consumption-based model, the empirical tests have to resolve all the specification issues and empirical difficulties encountered in the empirical CCAPM literature. Moreover, it is not clear why adding a nontrivial production sector into the CCAPM will necessarily bring us a greater empirical success. Once the model is rejected, the rejection may be due to the rejection of PCAPM specifically, or due to the rejection of preference restriction, or both. Therefore, the general equilibrium approach is subject to greater specification problems. On the other hand, the partial equilibrium approach only focuses on the producer's optimization problem and relates asset returns to the firm's marginal rates of transformation. This approach does not require the specification of the rest of the economic environment, and thus is less restrictive than its general equilibrium counterpart. However, imposing fewer restrictions comes at a cost. The partial equilibrium approach does not present a structural explanation as to why and how the factors affecting physical investment returns also affect equity returns. Hence, once the model is rejected, it is difficult to isolate the source of the rejection.

2.2.1 The General Equilibrium Test

Sharathchandra (1993) is the only empirical study in the literature adopting a general equilibrium approach. The model studied in his paper is a simplified version of Brock's (1982) general equilibrium model. The first-order conditions relate both the equity return and the physical investment return to the IMRS. To derive a testable formula, Sharathchandra further assumes that the representative consumer's preferences can be described by logarithmic utility. With logarithmic utility, the consumer always consumes a constant proportion of her total wealth. It can be shown that, in equilibrium, the stochastic discount factor (or the IMRS) is equivalent to the inverse of the equity return.

Therefore the firm's first order condition (also called Euler equation) can be expressed as $E_t \left[\frac{1}{R^E(t+1)} R^I(t+1) \right] = 1$, where R^E and R^I represent the equity return and the physical investment return, respectively. The physical investment return is expressed as a function of the unknown production parameter. The above Euler equation is then estimated and tested by means of the GMM procedure. Using the quarterly equity return and physical capital investment data for the entire U.S. economy from 1948 to 1990, the author concludes that the model can not be rejected. However, the supporting evidence provided by Sharathchandra should be interpreted with caution. First, the empirical test is based on a very restrictive assumption on the consumer's preferences, and thus may be sensitive to misspecification. Second, the model ignores the existence of adjustment costs. As argued in Cochrane (1991), adjustment costs are necessary to produce a time series variation in physical investment returns similar to that in stock returns.

2.2.2 The Partial Equilibrium Tests

For the reason pointed out at the beginning of this section, the empirical literature about the PCAPM has been dominated by partial equilibrium approaches. Broadly speaking, the partial equilibrium tests can be classified into two groups. One group of tests focus on the time series relationship between physical investment returns and equity returns. These studies investigate the hypothesis that the physical investment return should be equal to the equity return in every state of the world. The representative studies include Cochrane (1991), Bakshi, Chen, and Naka (1995), and Porter (1999). The other group of tests focuses on the pricing relationship between physical investment returns and equity returns. Particularly, they examine whether the physical investment returns contain sufficient information to correctly price the corresponding equity returns. The representative studies include Cochrane (1996) and Kasa (1997).

Cochrane (1991) studies the empirical linkage between the time-series variation in

stock returns and that in physical investment returns by using aggregate U.S. data from 1947 to 1987. Assuming complete markets and linear homogeneous production and adjustment cost functions, the producer's first-order condition implies that, in equilibrium, the firm's investment rate of return should be equal to its stock rate of return ex post, in every state of nature. Clearly, any attempt to directly test the above hypothesis is very challenging. Even if the theory is correct, the noise in the physical capital investment data will almost surely lead to the rejection of the hypothesis.

Instead of conducting a direct test, Cochrane exploits the following implication of the equivalence of physical investment returns and equity returns: If the null hypothesis is correct, then the coefficients in regressions of the equity returns and the physical investment returns on any set of variables should be equal. To empirically test this theoretical implication, Cochrane (1991) conducts three types of regression tests on the physical investment returns and the equity returns. The physical investment returns are constructed from gross fixed private domestic investment data, and the equity returns are computed as the gross rate of returns on CRSP value-weighted NYSE portfolio. The first test regresses current physical investment returns and stock returns on a set of forecasting variables dated in the past. These variables have been conventionally used in the literature to predict stock returns. The regression results show that the forecasts of physical investment returns and stock returns appear to be the same for most forecasting variables. The second test regresses the current physical investment and stock returns on the same set of forecasting variables dated in the future. The regression results show that both returns exhibit similar association with subsequent economic activity. As a final test, Cochrane regresses the two returns on past, contemporaneous, and subsequent investment/capital ratios. Although the two returns exhibit similar basic pattern of relationship with investment/capital ratios, the regression coefficients appear to be different from each other. In summary, Cochrane concludes that the physical investment returns and the stock returns are indeed closely linked to each other.

Bakshi, Chen, and Naka (1995) test a discrete-time PCAPM by using the quarterly Japanese physical investment return and stock return data from 1972 to 1990. The model studied is similar to the one in Cochrane (1991), except that the marginal product of capital is allowed to vary over time. Allowing for time-varying marginal product of capital implies that the time-variation in physical investment returns is driven by both investment/capital ratios and the marginal product of capital. Three types of empirical tests are carried out in the paper. Similar to Cochrane (1991), the first test regresses the physical investment returns and the stock returns on a common set of forecasting variable dated in the past. The estimated regression coefficients on each forecasting variable are found to be similar in magnitude for both regressions. The second test examines the predicting ability of the physical investment returns and the equity returns on future GNP growth and capital investment growth. The regression results show that both the past physical investment returns and the past equity returns have significant predicting power on future real activity. However, the forecasting ability of the equity returns is better than that of the physical investment returns. As a final test, the authors conduct an Euler equation-based test investigating whether the physical investment returns and the stock market returns are priced similarly. Similar to Sharathchandra (1993), the conditional moment condition involves using the inverse of the stock market return as the stochastic discount factor to price physical investment returns. The J-statistics from GMM estimation indicates that the model can not be rejected. Overall, the empirical findings provide supportive evidence for the PCAPM.

Porter (1999) is the only paper in the literature attempting to link industry physical investment returns to industry equity returns. The study modifies the model in Cochrane (1991) to include a planning phase for physical capital investment. The inclusion of the time-to-plan reflects the fact that there is a time lag between investment decision and the actual implementation of the investment. With this modification, equity returns are no longer a function of current investment expenditures, but are a function of cur-

rent investment decisions and the resulting future expenditures. Porter then applies the model to each of the 25 industries formed on the basis of two-digit and three-digit SIC code using annual physical investment and equity data from 1956 to 1996. Under the assumption of a constant time-to-plan across industries, Porter constructs physical investment return series adjusted for time-to-plan and equity return series adjusted for temporal aggregation. He finds that adjusting for time-to-plan and temporal aggregation greatly improves the correlation between the physical investment returns and the equity returns. Further, he regresses the equity portfolio returns on contemporaneous physical investment returns for each of the 25 industries, and finds that the equivalence relationship between the two return series can not be rejected for 16 out of 25 industries.

One important contribution of Porter's study is that time-to-plan and temporal aggregation may have nontrivial impacts in examining the time-series relationship between physical investment returns and equity returns. Moreover, Porter (1999) provides some useful insights into the linkage between industry physical investment returns and industry equity portfolio returns. However, the method adopted by Porter limits the study's ability to answer several deeper questions. First, although time-to-plan is a realistic consideration, and may be crucial in determining the correlation between physical investment returns and equity returns, it is also a parameter very difficult to estimate empirically. Assuming a constant time-to-plan across industries is not quite realistic, and it is not clear whether the estimated correlation is sensitive to different specifications of time-to-plan. Second, Porter's study ignores the interdependence of both equity returns and physical investment returns among different industries by conducting the empirical test one industry at a time. Such an approach makes the study unable to reveal the relative importance of an industry's physical investment return in pricing the cross-industry equity returns. Third, although empirical results provide some evidence for the equivalence of equity and physical investment returns, they also provide evidence for the rejection of the equivalence relationship. About 36 percent of the industries included in

the study reject the theory. Porter's study does not provide a rigorous statistical test for determining whether to reject the model when all the industries are considered.

Clearly, direct tests of the equivalence between physical investment returns and equity returns are very difficult to implement empirically. Theoretical conditions implying such equality (state by state) are also quite stringent (see Restoy and Rockinger (1994)). Market imperfections, noisy data, and approximations in production technologies are all likely to break up such equality. Hence, it is not surprising that about one third of the industries in Porter's study reject the null hypothesis despite all the efforts of adjusting for time-to-plan and temporal aggregation. Partly due to the above reasons, another group of partial equilibrium tests take a different route. They explore the pricing relationship between physical investment returns and equity returns by testing the absence of arbitrage or consistent pricing between the two sets of returns by constructing appropriate stochastic discount factors. The null hypothesis examined in these tests is less demanding and yet contains appealing implications on asset pricing. The rest of this section focuses on reviewing representative works along this line.

Cochrane (1996) is the first paper in the literature focusing on the pricing relationship between physical investment returns and equity returns. Specifically, Cochrane studies the validity of a factor pricing model. In the model, the physical investment returns are assumed to be the only pricing factors for the stock returns, i.e., the stochastic discount factor m is a linear combination of the physical investment returns only. The model is then estimated and tested by using the GMM procedure. Two types of physical investment returns are considered as factors for the stock returns: the gross private domestic nonresidential and residential investment returns. The stock returns studied include the returns for 10 portfolios of NYSE stocks sorted by market value. The GMM results suggest that the physical investment factor pricing model is rejected when only the excess equity returns are scaled by the selected instruments. However, the model can not be rejected when both the factors and the excess equity returns are scaled by

the selected instruments. Cochrane also compares the performance of this factor pricing model with several other popular asset pricing models based on the pricing errors of the mean excess returns. The physical investment factor pricing model performs at least as well as all the other models, and performs significantly better than the CCAPM and an ad hoc consumption growth factor model.

As an important contribution to the literature, Cochrane proposes an empirically implementable way to test whether physical investment returns contain enough information to correctly price equity returns. The proposed factor pricing model can be derived either by invoking no-arbitrage assumptions or by invoking appropriate preference assumptions in the general equilibrium framework of Brock (1982). Moreover, the estimation and testing of the factor model readily fits into the GMM framework proposed by Hansen and Singleton.

One shortcoming of Cochrane's study is that the study does not provide any formalized rule on selecting the number and nature of the intertemporal production technologies that drive equity returns. Instead, Cochrane uses two arbitrarily selected aggregate technologies (nonresidential investment and residential investment) to construct the physical investment return series. Clearly, such classification is based on the usage of the final product rather than the characteristics of the production process. Hence, these two aggregated production technologies may not perform well in terms of capturing all of the intertemporal investment opportunities in the economy. As a matter of fact, the model is rejected when only the excess equity returns are scaled by the selected instrument. Part of the reason for the rejection may be due to the insufficient spanning ability of the physical investment returns constructed from the selected aggregate production technologies. This highlights a joint hypothesis test problem embedded in Cochrane's approach. Both the factor pricing model and the aggregation of production technologies are modeling assumptions. Once the model is rejected, it is very difficult to identify whether the rejection comes from the physical investment factor pricing model per se or

from the inappropriate aggregation of production technologies.

Kasa (1997) compares the ability of the CCAPM and the PCAPM to explain the cross-country and time-series variation of stock returns in the United States, Japan, the United Kingdom, Germany, and Canada. Following Cochrane (1996), Kasa focuses on the performances of two factor pricing models. For the CCAPM, the factor is defined as the population-weighted world consumption growth rate. For the PCAPM, the factor is defined as the population-weighted world investment/capital growth rate. Here, the growth in each country's investment-capital ratio is used as a proxy for the stochastic component of each country's physical investment return. For asset returns, the time series of each country's aggregate stock returns are used in the study. Both the consumption and physical investment factor pricing models are estimated and tested by means of the GMM procedure. The empirical results suggest that neither the PCAPM nor the CCAPM can be rejected by the data. However, the PCAPM performs significantly better than the CCAPM in explaining the cross-country variation in stock returns.

In summary, the empirical research on the PCAPM is still at an early stage of development. Researchers are still searching for satisfactory methods to empirically test the theoretical implications of the PCAPM. In this paper, I extend the research by Cochrane (1996) and Porter (1999) by proposing an alternative approach to identifying the linkage between physical investment returns and equity returns using cross-industry data. Hopefully, it will provide additional insights into the theoretical implications of the PCAPM.

3. THE FIRM'S OPTIMIZATION PROBLEM

Unlike equity returns, physical investment returns do not have actual market quotes, and have to be derived by solving the profit maximization problem of the representative firm. In this chapter, I use the dynamic programming technique to solve the firm's intertemporal optimization problem and to derive a closed-form representation for the physical investment return.

Consider a representative firm that produces a single type of output with a single type of capital input. The objective of the firm is to maximize the expected discounted net cash flow by selecting the optimal physical capital investment plan.

The firm solves the following optimization problem:

$$\max_{\{I_t\},\{L_t\}} E_t\{\sum_{h=0}^{\infty} m_{t,t+h}\Pi_{t+h}\} \tag{3.1}$$

subject to

$$\Pi_t \equiv p_t Y_t - I_t - A(I_t, K_t) - \omega_t L_t, \tag{3.2}$$

$$Y_t \equiv \epsilon_t F(K_t, L_t), \tag{3.3}$$

$$K_{t+1} \equiv (1 - \delta)K_t + I_t, \tag{3.4}$$

where the t subscript denotes time, m is the equilibrium stochastic discount factor, Π_t denotes the firm's net cash flow, K_t denotes the firm's beginning physical capital stock, I_t denotes the new physical capital investment, L_t denotes labor input, Y_t denotes the firm's output, and p_t, ϵ_t, ω_t and δ denote the output price, a random productivity shock, the wage rate, and the depreciation rate, respectively. The production function $F(\cdot)$

is assumed to be continuously differentiable, strictly increasing, and strictly concave on R^+ with $F(0, L_t) = 0$, $F'(0, L_t) = \infty$, and $F'(\infty, L_t) = 0$. The technology shock $\epsilon_t \geq 0$ is assumed to be independently, identically distributed with stationary probability distribution. The adjustment cost function $A(\cdot)$ reflects the cost of investment beyond the purchase price of capital goods. It has the properties that $A(\cdot) \geq 0$, $\partial A/\partial I_t > 0$, and $\partial A/\partial K_t < 0$. Finally, the motion of the capital stock is given by equation (3.4).

To solve the above problem, let us substitute equations (3.2) and (3.3) into the objective function and define

$$
\begin{aligned}
\tilde{V}_t &\equiv \max E_t\{\sum_{h=0}^{\infty} m_{t,t+h}[p_{t+h}\epsilon_{t+h}F(K_{t+h}, L_{t+h}) - I_{t+h} - A(I_{t+h}, K_{t+h}) - \omega_{t+h}L_{t+h}]\} \\
&= \max p_t \epsilon_t F(K_t, L_t) - I_t - A(I_t, K_t) - \omega_t L_t + \\
&\quad E_t\{\sum_{h=1}^{\infty} m_{t,t+h}[p_{t+h}\epsilon_{t+h}F(K_{t+h}, L_{t+h}) - I_{t+h} - A(I_{t+h}, K_{t+h}) - \omega_{t+h}L_{t+h}]\}. \quad (3.5)
\end{aligned}
$$

Taking derivative with respect to I_t gives

$$
0 = -1 - A_I(t) + E_t\{\sum_{h=1}^{\infty} m_{t,t+h}[p_{t+h}\epsilon_{t+h}F_K(t+h) - A_K(t+h)]K_I(t+h)\} \quad (3.6)
$$

Here $A_I(t)$ denotes the partial derivative with respect to I_t, evaluated with respect to the appropriate arguments at time t. Similar interpretation can be given to $F_K(t+h)$, $A_K(t+h)$, and $K_I(t+h)$.

The equation of the motion of capital stock in (3.4) implies that

$$
\begin{aligned}
K_{t+h} &= (1-\delta)K_{t+h-1} + I_{t+h-1} \\
&= (1-\delta)[(1-\delta)K_{t+h-2} + I_{t+h-2}] + I_{t+h-1} \\
&= (1-\delta)^2 K_{t+h-2} + (1-\delta)I_{t+h-2} + I_{t+h-1} \\
&= \ldots \\
&= (1-\delta)^h K_t + (1-\delta)^{h-1}I_t + \sum_{n=1}^{h-1}(1-\delta)^{n-1}I_{t+h-n}. \quad (3.7)
\end{aligned}
$$

Taking derivative with respect to I_t gives

$$
K_I(t+h) \equiv \frac{\partial K_{t+h}}{\partial I_t} = (1-\delta)^{h-1}. \quad (3.8)
$$

Substituting equation (3.8) into equation (3.6) gives

$$1 + A_I(t) = E_t\{\sum_{h=1}^{\infty} m_{t,t+h}(1 - \delta)^{h-1}[p_{t+h}\epsilon_{t+h}F_K(t + h) - A_K(t + h)]\}. \tag{3.9}$$

Using the fact that $m_{t,t+h+1} = m_{t,t+1} * m_{t+1,t+h+1}$ (Cochrane (1996)), equation (3.9) can be rewritten as

$$1 + A_I(t) = E_t\{m_{t,t+1}[p_{t+1}\epsilon_{t+1}F_K(t + 1) - A_K(t + 1)]\} + E_t\{m_{t,t+1}\sum_{h=1}^{\infty} m_{t+1,t+h+1}$$
$$(1 - \delta)^h[p_{t+h+1}\epsilon_{t+h+1}F_K(t + h + 1) - A_K(t + h + 1)]\}. \tag{3.10}$$

Forwarding equation (3.9) by one period implies

$$1 + A_I(t + 1) = E_{t+1}\{\sum_{h=1}^{\infty} m_{t+1,t+h+1}(1 - \delta)^{h-1}$$
$$[p_{t+h+1}\epsilon_{t+h+1}F_K(t + h + 1) - A_K(t + h + 1)]\}. \tag{3.11}$$

Substituting equation (3.11) into equation (3.10) gives

$$1 + A_I(t) = E_t\{m_{t,t+1}[p_{t+1}\epsilon_{t+1}F_K(t + 1) - A_K(t + 1)]\} +$$
$$E_t\{m_{t,t+1}(1 - \delta)[1 + A_I(t + 1)]\}, \tag{3.12}$$

or, equivalently,

$$E_t\left[m_{t,t+1}R^I(t + 1)\right] = 1, \tag{3.13}$$

where

$$R^I(t + 1) \equiv \frac{p_{t+1}\epsilon_{t+1}F_K(t + 1) - A_K(t + 1) + (1 - \delta)[1 + A_I(t + 1)]}{1 + A_I(t)}. \tag{3.14}$$

Equation (3.14) can be interpreted as the firm's marginal rate of return on physical investment. As pointed out by Cochrane (1996), "the investment return is the marginal rate at which a firm can transfer resources through time by increasing investment today and decreasing it at a future date, leaving its production plan unchanged at all other dates." To invest an additional unit of capital at time t, the firm has to sell less and to bear a certain amount of adjustment cost. The denominator $1 + A_I(t)$ captures this effect.

The numerator in the definition of $R^I(t+1)$ represents the marginal benefit realized at time $t+1$ from the additional capital investment at time t. The term $\epsilon_{t+1}F_K(t+1)$ is the extra output produced from the additional investment at time t, while $A_K(t+1)$ is the effect of the additional capital investment on the adjustment cost at time $t+1$. To maintain its production plan unchanged at all other dates, the firm has to lower its investment at time $t+1$. The decrease of capital investment allows the firm to sell more and to reduce its adjustment cost. The term $(1-\delta)[1+A_I(t+1)]$ captures these positive effects on time $t+1$ profit. In summary, $R^I(t+1)$ is just the ratio of marginal benefit at time $t+1$ over marginal cost of one additional unit of capital investment at time t, and thus is a legitimate definition of physical investment return.

For the purpose of constructing physical investment series, I make the following assumptions about the production function and the adjustment cost function:

$$Y_t \equiv \epsilon_t F(K_t, L_t) \;=\; \epsilon_t K_t^\alpha L_t^{1-\alpha}, \tag{3.15}$$

$$A(I_t, K_t) \;=\; \frac{\eta}{2}\left(\frac{I_t}{K_t}\right)^2 K_t. \tag{3.16}$$

The production function has the standard Cobb-Douglas form. The adjustment cost function adopted here is referred in the literature as the symmetric convex (quadratic) cost function. It is consistent with the basic assumption made by the literature on the q-theory of investment since it is linear homogeneous of degree zero in I_t and K_t. Further, the functional form (3.16) imposes symmetry around $I_t/K_t = 0$, so that the adjustment cost of increasing K_t by a certain percent is equal to that of a similar-size cut in K_t. Later on I will relax the assumption of symmetry and allow for asymmetric adjustment costs. The parameter η in equation (3.16) is called the adjustment cost coefficient. For a given size of new capital investment and current capital stock, a firm with higher η will incur higher adjustment cost than a firm with lower η. Hence, η measures the relative cost for a firm to adjust its capital stock. If two firms engaging in the same production activity differ only in the level of η, then the firm with higher η (higher adjustment cost)

adjusts its capital stock in a more sluggish manner than the firm with lower η (lower adjustment cost) in response to a productivity shock.

Equations (3.15) and (3.16) imply that

$$\epsilon_t F_K(t) = \alpha \left(\frac{Y_t}{K_t} \right), \tag{3.17}$$

$$A_I(t) = \eta \left(\frac{I_t}{K_t} \right), \tag{3.18}$$

$$A_K(t) = -\frac{\eta}{2} \left(\frac{I_t}{K_t} \right)^2. \tag{3.19}$$

Substituting equation (3.17), equation (3.18), and equation (3.19) into the definition of $R^I(t+1)$ gives

$$R^I(t+1) = \frac{\alpha \left(\frac{p_{t+1} Y_{t+1}}{K_{t+1}} \right) + \frac{\eta}{2} \left(\frac{I_{t+1}}{K_{t+1}} \right)^2 + (1 - \delta) \left[1 + \eta \left(\frac{I_{t+1}}{K_{t+1}} \right) \right]}{1 + \eta \left(\frac{I_t}{K_t} \right)}. \tag{3.20}$$

Equation (3.20) will be used to compute physical investment returns in the following empirical tests.

4. ESTIMATION AND TESTING METHODS

In this chapter, I present the estimation and testing methods used to examine the validity of the spanning assumption. The proposed nonparametric procedure is based on entropic principles and no-arbitrage constraints. For detailed illustration on the basic entropic principles and no-arbitrage constraints, please refer to appendices A and B.

4.1 The Physical Investment Factor Pricing Model and the Spanning Assumption

Most asset pricing models in the finance literature focus exclusively on the payoff space of financial securities. Within the context of PCAPM, one essentially expands the payoff space to include that of physical investment. After constructing the physical investment return series according to equation (3.20), one can then examine the pricing relationship between physical investment returns and equity returns.

If there are no arbitrage opportunities in the expanded payoff space including both the physical investment and the financial investment, then the Fundamental Theorem of Asset Pricing and the Pricing Rule Representation Theorem (Dybvig and Ross (1992)) imply that a stochastic discount factor m exists such that $m \gg 0$ and

$$E(mR_i^I) \quad = \quad 1, \qquad \forall i, \tag{4.1}$$

$$E(mR_j^E) \quad = \quad 1, \qquad \forall j. \tag{4.2}$$

Here. R_i^I denotes the physical investment return on production technology i, and R_j^E

denotes the financial return on equity j. Intuitively, physical investment is just an alternative way of transforming goods across dates and states. Therefore, the physical investment returns should be priced by the same discount factor m that correctly prices the equity returns.

Equations (4.1) and (4.2) are empirically testable only when specific restrictions are imposed on m. What form m should take is one of the central topics in empirical asset pricing. The law of one price implies that there always exists a discount factor m that is a linear combination of the physical investment returns and the equity returns and that prices both (Chamberlain and Rothschild (1983)). Mathematically, there exist bs such that

$$m = \sum_i b_i R_i^I + \sum_j b_j R_j^E, \tag{4.3}$$

and that equations (4.1) and (4.2) are satisfied. However, this m may be negative, which implies that dominant trading strategies (and obviously arbitrage opportunities) exist in this economy.

Cochrane (1996) imposes additional restrictions on equation (4.3), and proposes the following physical investment factor pricing model:

$$m = \sum_i b_i R_i^I. \tag{4.4}$$

Note that the equity returns are completely excluded from the stochastic discount factor. In other words, the above factor pricing model uses only the physical investment returns as factors to price equity returns. One critical assumption justifying the existence of stochastic discount factor (4.4) is that financial securities offer no additional spanning opportunities on the payoff space beyond those offered by physical capital investment. This is just a restatement of the *spanning assumption* discussed in the introduction.

One thing to point out is that the validity of (4.4) is not a direct implication from a pure PCAPM. A pure PCAPM imposes no restrictions on the space of equity returns, and reads any equity return off a producer's first-order conditions. With general time-

separable preferences, the stochastic discount factor implied by a pure PCAPM is in general a nonlinear function of the physical investment returns. The physical investment factor pricing model can be derived either by imposing arbitrage assumptions or by imposing preference and technology assumptions. If one assumes that the stocks traded in NYSE are claims to different combinations of N production technologies, then no-arbitrage between the physical investment market and the financial market implies that the spanning assumption and the physical investment factor pricing model must hold. Alternatively, within the context of the Brock-style general equilibrium model, sufficient conditions for the validity of the spanning assumption and the linearity of m require certain restrictions on technology and preferences. In the example given by Cochrane (1996), the general equilibrium PCAPM is characterized by the standard one-sector stochastic growth model with log utility, Cobb-Douglas production, and full depreciation. In this special case one can show that the stochastic discount factor can be first-order approximated by a linear function of the physical investment return.

Our goal is to test whether the physical investment returns contain sufficient information to correctly price the corresponding equity returns. Two approaches can be taken to conduct the empirical test. First, one can examine the validity of the physical investment factor pricing model (4.4). I refer to this approach as *the linear factor pricing approach*. Cochrane (1996) selects two aggregate production technologies to construct m, and examines whether such a stochastic discount factor is able to capture the relative risk structure of equity portfolios of different market capitalization. Kasa (1997) follows a similar approach to examining the performance of the physical investment factor pricing model in explaining the international variations in equity returns. The linear factor pricing approach offers a tractable way to empirically test the PCAPM since it readily fits into the GMM framework. However, such an approach has an undesirable property that the stochastic discount factor may take on negative values, which is inconsistent with the spirit of no-arbitrage. The second approach involves directly testing the validity

of the spanning assumption. Instead of assuming a parametric form for the stochastic discount factor, the proposed testing procedure focuses on recovering and comparing the state price densities for the physical investment returns and the equity returns. Hence, I refer to this approach as *the state price density approach*. In the following sections, I focus on discussing the testing procedure, data construction, empirical results, and the relationship between the state price density approach and the linear factor pricing approach.

4.2   Procedures for Testing the Validity of the Spanning Assumption

In this section, I present a three-step procedure for testing the spanning assumption based on entropic principles and no-arbitrage constraints.

Let R be the gross rate of return for any asset. Then the no-arbitrage constraint implies that there exists $m \gg 0$ such that

$$E\left[mR\right] = 1. \tag{4.5}$$

Dividing both sides by $E(m)$ and assuming the existence of a risk free rate r, we can rewrite equation (4.5) as

$$E\left[\frac{m}{E[m]}R\right] = \int \frac{m}{E[m]}R d\pi = r, \tag{4.6}$$

where $d\pi$ denotes the actual probability measure over the states of the world. Utilizing the change of measure

$$d\pi_{rn} = \frac{m}{E[m]}d\pi, \tag{4.7}$$

we can express equation (4.6) in the following equivalent form:

$$E_{\pi_{rn}}\left[\frac{1}{r}R\right] = 1. \tag{4.8}$$

Equation (4.8) is the risk-neutral representation of the no-arbitrage constraint, in which $d\pi_{rn}$ is referred to as the *risk-neutral probability measure*, and $d\pi_{rn}/d\pi$ is called the *state price probability density* (SPD). Note that, unless the market is complete, the risk-neutral probability (or the SPD) will not be unique.

Assuming that no arbitrage opportunities exist separately in either the physical investment market or the stock market, equation (4.8) implies the following two no-arbitrage constraints (one for each market):

$$E_{\pi_I}\left[\frac{1}{r}R_i^I\right] = 1, \qquad i = 1, 2, \ldots, M, \qquad (4.9)$$

$$E_{\pi_E}\left[\frac{1}{r}R_j^E\right] = 1, \qquad j = 1, 2, \ldots, N. \qquad (4.10)$$

Here $d\pi_I$ and $d\pi_E$ denote the risk-neutral probability measures for the physical investment market and the stock market respectively, R_i^I denotes the one-period gross return of investing one dollar capital in production technology i, and R_j^E denotes the one-period gross return of investing one dollar in equity or portfolio j in the stock market.

The spanning assumption states that the payoff space of physical capital investment spans that of financial investment in the stock market. No-arbitrage constraints alone ensure that the two markets share at least one risk-neutral probability measure. If, in addition, the spanning assumption holds, then any risk-neutral probability measure correctly pricing the physical investment returns should also be able to price the equity returns. Based on the above implication, one natural way to test the spanning assumption is to examine whether the risk-neutral probability measure recovered from the physical investment returns is consistent with that recovered from the equity returns. Since

$$d\pi_I = d\pi_E \Longleftrightarrow \frac{d\pi_I}{d\pi} = \frac{d\pi_E}{d\pi}, \qquad (4.11)$$

it will suffice for us to recover and compare the SPDs in the two markets.

Clearly, how to recover and compare the SPDs (or risk-neutral probability measures) is crucial in conducting the empirical test. Fortunately, the entropic framework provides

handy ways to complete this task. In summary, the proposed testing procedure consists of the following three steps. Step 1 recovers the SPD from the physical investment return data. Step 2 uses the recovered SPD from step 1 as prior information, and recovers the SPD for the equity returns. Finally, step 3 compares the entropic distance between these two sets of SPDs. Under the spanning assumption, the entropic distance should equal zero.

For the remaining of this section, I will provide a detailed description on how to implement the above three steps.

4.2.1 Recovering the State Price Probability Density from the Physical Investment Return Data

To find the SPD correctly pricing the physical investment returns, I follow the canonical evaluation method proposed by Stutzer (1995). Let $\tilde{R}_i^I \equiv R_i^I/r$ be the discounted physical investment return on production technology i and M be the total number of different technologies in the economy. Then the SPD for physical investment returns can be recovered by solving the following optimization problem:

$$\min_{\pi_I} I(\pi_I, \pi) \equiv \int \log(\frac{d\pi_I}{d\pi}) d\pi_I \qquad (4.12)$$

subject to

$$E_{\pi_I}\left[\tilde{R}_i^I\right] = 1, \qquad i = 1, 2, \dots, M. \qquad (4.13)$$

$$\int d\pi_I = 1. \qquad (4.14)$$

In information theory, $I(\pi_I, \pi)$ is referred to as the *Kullback-Leibler Information Criterion* (KLIC). It is well known that $I(\pi_I, \pi) \geq 0$ with equality only when $d\pi_I = d\pi$.

The above procedure is well justified by Bayesian, information-theoretic econometrics. Before observing any return data, if one is reluctant to impose any arbitrary process

assumption, then it is reasonable to just assume that the unknown risk-neutral proba-
bility measure $d\pi_I$ is the same as the actual probability measure $d\pi$. After collecting the
return data and constructing the no-arbitrage constraints, one can then use the addi-
tional information to update this prior belief, and to formulate a posterior risk-neutral
probability measure. It is reasonable to require that the posterior probability measure
incorporate no additional information other than that contained in the no-arbitrage
constraints.

To formalize the above concept in a mathematical framework, we need to construct
a well rationalized criterion quantifying the amount of information gained in changing
from the actual probability measure $d\pi$ to the risk-neutral measure $d\pi_I$. The axiomatic
rationalization for using the KLIC to measure the information gain was provided by
Khinchin (1957) and Hobson (1971). Khinchin (1957) considers a special case of the
KLIC when $d\pi$ is a discrete uniform distribution. In this case, the minimization of I is
equivalent to the maximization of the Shannon entropy $-\sum_s \pi_I(s) \log(\pi_I(s))$, where s
represents the state of the world. In an information theoretic context, Shannon entropy
is used to measure the amount of uncertainty embodied in a probability distribution.
Khinchin then formulates intuitively appealing axioms that a measure of information
uncertainty should satisfy, and shows that the Shannon entropy is the unique measure
satisfying the axioms.

Hobson (1971) generalizes Khinchin's uniqueness theorem and considers the general
case that the actual probability measure is not uniformly distributed. For any arbitrary
$d\pi$, the measure of information gain from $d\pi$ to $d\pi_I$ should satisfy the following axioms:

1. Any information gain function should be a continuous function of its argument, so
   that the information changes only a small amount when the probabilities change
   by a small amount.

2. A mere relabeling of the states (i.e., which of the possible returns is dubbed the

first possible return, the second possible return, etc.) should not change the value of I.

3. No information is gained unless there is a change of probability measure, i.e., $I(\pi, \pi) = 0$.

4. Suppose that $d\pi$ is uniformly distributed on a subset of m outcomes (zero elsewhere), and $d\pi_I$ is also uniformly distributed, but on only n of those outcomes, $n \leq m$. Then I should be increasing in m, because more information is gained when $d\pi_I$ rules out more of the outcomes possible under $d\pi$. Furthermore, it is required that I should be decreasing in n, as less information is gained when $d\pi_I$ is more diffuse.

5. Any information gain function should satisfy a "composition rule". The details of the rule is omitted here. Interesting readers should refer to Hobson(1971).

Hobson then shows that the only functions satisfying the above axioms are proportional to the KLIC defined in equation (4.12).

It is well-known that the solution to problem (4.12) is attained by the following strictly positive, generalized exponential density, usually called a *Gibbs* density (see Appendix A for proof):

$$\frac{d\pi_I^*}{d\pi} = \frac{\exp[\sum_{i=1}^{M} \bar{\lambda}_i^I (\bar{R}_i^I - 1)]}{E_\pi\{\exp[\sum_{i=1}^{M} \bar{\lambda}_i^I (\bar{R}_i^I - 1)]\}}, \tag{4.15}$$

where the parameter vector $\bar{\lambda}^I$ can be found by solving the following convex minimization problem:

$$\bar{\lambda}^I \equiv arg\min_{\lambda^I} \Omega(\lambda^I), \tag{4.16}$$

where

$$\Omega(\lambda^I) \equiv E_\pi\{\exp[\sum_{i=1}^{M} \lambda_i^I (\bar{R}_i^I - 1)]\}. \tag{4.17}$$

Substituting equation (4.15) into the objective function gives us the minimized KLIC criterion:

$$I(\pi_I^*, \pi) = -\log \Omega(\tilde{\boldsymbol{\lambda}}^I).$$ (4.18)

Given a time series of physical investment returns, we may estimate the solution to (4.16) by substituting a time average for the expectation operator. Such a substitution can be justified by the Law of Large Numbers since the expectation is taken with respect to the actual probability distribution over the states of the world. Thus,

$$\hat{\boldsymbol{\lambda}}^I \equiv arg \min_{\boldsymbol{\lambda}^I} \hat{\Omega}(\boldsymbol{\lambda}^I),$$ (4.19)

where

$$\hat{\Omega}(\boldsymbol{\lambda}^I) \equiv \frac{1}{T} \sum_{t=1}^{T} \exp[\sum_{i=1}^{M} \lambda_i^I(\tilde{R}_{it}^I - 1)].$$ (4.20)

Substituting $\hat{\boldsymbol{\lambda}}^I$ into equation (4.15) gives us the estimated SPD $d\hat{\pi}_I^*/d\pi$ for the physical investment returns with

$$\frac{d\hat{\pi}_I^*}{d\pi} = \frac{\exp[\sum_{i=1}^{M} \hat{\lambda}_i^I(\tilde{R}_i^I - 1)]}{E_\pi\{\exp[\sum_{i=1}^{M} \hat{\lambda}_i^I(\tilde{R}_i^I - 1)]\}}.$$ (4.21)

Note that the specific form of the estimated risk-neutral measure $d\hat{\pi}_I^*$ for physical investment returns depends on our assumption about the actual probability distribution $d\pi$. In many applications, one may need to explicitly define $d\pi$. One frequently made assumption is that $d\pi$ is uniformly distributed. As we shall see in step 2, such an assumption about the actual probability distribution over the states of the world is not necessary for the purpose of testing the spanning assumption. It will suffice to proceed with the estimated SPD $d\hat{\pi}_I^*/d\pi$. In other words, the proposed testing procedure is not sensitive to different specifications about the actual probability measure.

4.2.2 Recovering the State Price Density for Equity Returns Using $d\hat{\pi}_I^*/d\pi$ as Prior Information

Let $\tilde{R}_j^E \equiv R_j^E/r$ be the discounted gross rate of return for equity or portfolio j. Assume that there are a total of N equities or portfolios in the stock market. Using $d\hat{\pi}_I^*/d\pi$ as the prior information, we can recover the SPD for equity returns by solving the following optimization problem:

$$\min_{\pi_E} I(\pi_E, \hat{\pi}_I^*) \equiv \int \log(\frac{d\pi_E}{d\hat{\pi}_I^*})d\pi_E \tag{4.22}$$

subject to

$$E_{\pi_E}[\tilde{R}_j^E] = 1, \qquad j = 1, 2, \ldots, N, \tag{4.23}$$

$$\int d\pi_E = 1. \tag{4.24}$$

The Bayesian interpretation of the KLIC $I(\pi_E, \hat{\pi}_I^*)$ is similar to that of $I(\pi_I, \pi)$ in step 1. Before gathering any equity return data, one may believe that the equity market and the physical investment market share the same risk-neutral probability measure (or SPD). After observing the equity returns and constructing the no-arbitrage constraints, one can then update her prior belief and formulate a posterior risk-neutral probability measure for the equity returns. By minimizing the KLIC $I(\pi_E, \hat{\pi}_I^*)$ subject to no-arbitrage constraints, one makes sure that the updating process incorporates no additional information other than that contained in the no-arbitrage constraints.

The solution to the above problem is the following Gibbs density:

$$\frac{d\pi_E^*}{d\pi} = \frac{\exp[\sum_{j=1}^N \tilde{\lambda}_j^E(\tilde{R}_j^E - 1)]}{E_{\hat{\pi}_I^*}\{\exp[\sum_{j=1}^N \tilde{\lambda}_j^E(\tilde{R}_j^E - 1)]\}} \frac{d\hat{\pi}_I^*}{d\pi}, \tag{4.25}$$

where the parameter vector $\tilde{\lambda}^E$ is determined by

$$\tilde{\lambda}^E \equiv arg \min_{\lambda^E} \Omega(\lambda^E), \tag{4.26}$$

where

$$\Omega(\boldsymbol{\lambda}^E) \equiv E_{\hat{\pi}_I^*}\{\exp[\sum_{j=1}^N \lambda_j^E(\tilde{R}_j^E - 1)]\}. \tag{4.27}$$

Since the expectation in equation (4.27) is taken with respect to the risk-neutral probability measure $\hat{\pi}_I^*$, the sample time average is no longer the consistent estimator. To empirically estimate $\tilde{\boldsymbol{\lambda}}^E$ I perform the following change of measure:

$$\begin{aligned}
\Omega(\boldsymbol{\lambda}^E) &\equiv E_{\hat{\pi}_I^*}\{\exp[\sum_{j=1}^N \lambda_j^E(\tilde{R}_j^E - 1)]\} \\
&= E_\pi\{\exp[\sum_{j=1}^N \lambda_j^E(\tilde{R}_j^E - 1)]\frac{d\hat{\pi}_I^*}{d\pi}\}.
\end{aligned} \tag{4.28}$$

Substituting the estimated SPD for physical investment returns into the above equation gives

$$\begin{aligned}
\Omega(\boldsymbol{\lambda}^E) &= E_\pi\{\exp[\sum_{j=1}^N \lambda_j^E(\tilde{R}_j^E - 1)]\frac{\exp[\sum_{i=1}^M \hat{\lambda}_i^I(\tilde{R}_i^I - 1)]}{E_\pi\{\exp[\sum_{i=1}^M \hat{\lambda}_i^I(\tilde{R}_i^I - 1)]\}}\} \\
&= \frac{E_\pi\{\exp[\sum_{j=1}^N \lambda_j^E(\tilde{R}_j^E - 1) + \sum_{i=1}^M \hat{\lambda}_i^I(\tilde{R}_i^I - 1)]\}}{E_\pi\{\exp[\sum_{i=1}^M \hat{\lambda}_i^I(\tilde{R}_i^I - 1)]\}}.
\end{aligned} \tag{4.29}$$

Since all the expectations in equation (4.29) are taken with respect to the actual probability measure, we can consistently estimate $\tilde{\boldsymbol{\lambda}}^E$ by substituting time averages for the expectation operators. Hence,

$$\hat{\boldsymbol{\lambda}}^E \equiv arg \min_{\boldsymbol{\lambda}^E} \hat{\Omega}(\boldsymbol{\lambda}^E), \tag{4.30}$$

where

$$\hat{\Omega}(\boldsymbol{\lambda}^E) \equiv \frac{1}{T}\sum_{t=1}^T \left\{ \frac{\exp[\sum_{i=1}^M \hat{\lambda}_i^I(\tilde{R}_{it}^I - 1)]}{\frac{1}{T}\sum_{t=1}^T \exp[\sum_{i=1}^M \hat{\lambda}_i^I(\tilde{R}_{it}^I - 1)]} \exp[\sum_{j=1}^N \lambda_j^E(\tilde{R}_{jt}^E - 1)] \right\}. \tag{4.31}$$

For notation purposes, define

$$Q_T(\tilde{R}, \boldsymbol{\lambda}^E) \equiv \sum_{t=1}^T \left\{ \frac{\exp[\sum_{i=1}^M \hat{\lambda}_i^I(\tilde{R}_{it}^I - 1)]}{\frac{1}{T}\sum_{t=1}^T \exp[\sum_{i=1}^M \hat{\lambda}_i^I(\tilde{R}_{it}^I - 1)]} \exp[\sum_{j=1}^N \lambda_j^E(\tilde{R}_{jt}^E - 1)] \right\}. \tag{4.32}$$

Then equation (4.30) can be rewritten as

$$\hat{\boldsymbol{\lambda}}^E \equiv arg \min_{\boldsymbol{\lambda}^E} \frac{1}{T}Q_T(\tilde{R}, \boldsymbol{\lambda}^E). \tag{4.33}$$

Substituting the estimated $\hat{\boldsymbol{\lambda}}^E$ into equation (4.25) gives us the estimated SPD $d\hat{\pi}_E^*/d\pi$ for the equity returns with

$$\frac{d\hat{\pi}_E^*}{d\pi} = \frac{\exp[\sum_{j=1}^{N} \hat{\lambda}_j^E(\tilde{R}_j^E - 1)]}{E_{\hat{\pi}_I^*}\{\exp[\sum_{j=1}^{N} \hat{\lambda}_j^E(\tilde{R}_j^E - 1)]\}} \frac{d\hat{\pi}_I^*}{d\pi}. \tag{4.34}$$

4.2.3 Testing the Null Hypothesis that the Two State Price Densities Are Identical

If the spanning assumption holds, then any risk-neutral probability measure correctly pricing physical investment returns should also be able to price the corresponding equity returns. In other words, the two SPDs recovered in step 1 and step 2 should not be significantly different from each other. Note from equation (4.25) that

$$\frac{d\pi_E^*}{d\pi} = \frac{d\hat{\pi}_I^*}{d\pi} \iff \bar{\boldsymbol{\lambda}}^E = \mathbf{0}. \tag{4.35}$$

Therefore, the null hypothesis for testing the equivalence between the two SPDs can be stated as

$$H_0 : \bar{\lambda}_j^E = 0, \qquad j = 1, 2, \dots, N. \tag{4.36}$$

To carry out the above hypothesis test, we need first to study the asymptotic properties of $\hat{\boldsymbol{\lambda}}^E$, the sample estimate for $\bar{\boldsymbol{\lambda}}^E$. Note that the estimator $\hat{\boldsymbol{\lambda}}^E$ is an extremum estimator. Theorems 4.1.2 and 4.1.3 in Amemiya (1985, pp. 110-111) characterize regularity conditions for an extremum estimator to be consistent and asymptotically normally distributed. Stutzer (1995, pp. 381) discusses reasonable assumptions under which the regularity conditions hold in our context. Under these regularity conditions, $\hat{\boldsymbol{\lambda}}^E$ is a consistent estimator for $\bar{\boldsymbol{\lambda}}^E$, and has the following asymptotic normal distribution:

$$\sqrt{T}(\hat{\boldsymbol{\lambda}}^E - \bar{\boldsymbol{\lambda}}^E) \longrightarrow N\left[0, H(\bar{\boldsymbol{\lambda}}^E)^{-1}B(\bar{\boldsymbol{\lambda}}^E)H(\bar{\boldsymbol{\lambda}}^E)^{-1}\right], \tag{4.37}$$

where

$$H(\bar{\boldsymbol{\lambda}}^E) \equiv \lim \frac{1}{T}E\left(\frac{\partial^2 Q_T}{\partial \boldsymbol{\lambda}^E \partial \boldsymbol{\lambda}^{E\prime}}\right)_{\bar{\boldsymbol{\lambda}}^E}, \tag{4.38}$$

$$B(\tilde{\boldsymbol{\lambda}}^E) \equiv \lim \frac{1}{T} E \left(\frac{\partial Q_T}{\partial \boldsymbol{\lambda}^E} \frac{\partial Q_T}{\partial \boldsymbol{\lambda}^{E\prime}} \right)_{\tilde{\boldsymbol{\lambda}}^E}. \tag{4.39}$$

Here, the subscript $\tilde{\boldsymbol{\lambda}}^E$ indicates that both $H(\cdot)$ and $B(\cdot)$ are evaluated at the true parameter value $\tilde{\boldsymbol{\lambda}}^E$. The above asymptotic normality and the null hypothesis imply that

$$T \hat{\boldsymbol{\lambda}}^{E\prime} \left[\hat{H}_T^{-1} \hat{B}_T \hat{H}_T^{-1} \right]^{-1} \hat{\boldsymbol{\lambda}}^E \longrightarrow \chi_N^2. \tag{4.40}$$

Here, $\hat{H}_T$ and $\hat{B}_T$ are consistent estimators for $H(\tilde{\boldsymbol{\lambda}}^E)$ and $B(\tilde{\boldsymbol{\lambda}}^E)$ respectively. In particular, $\hat{H}_T$ can be calculated by evaluating the Hessian matrix of $T^{-1} Q_T(\tilde{R}, \boldsymbol{\lambda}^E)$ at $\hat{\boldsymbol{\lambda}}^E$, and $\hat{B}_T$ is just the covariance matrix of the gradient of

$$\frac{\exp[\sum_{i=1}^M \hat{\lambda}_i^I (\tilde{R}_{it}^I - 1)]}{T^{-1} \sum_{t=1}^T \exp[\sum_{i=1}^M \hat{\lambda}_i^I (\tilde{R}_{it}^I - 1)]} \exp[\sum_{j=1}^N \lambda_j^E (\tilde{R}_{jt}^E - 1)], \tag{4.41}$$

evaluated at $\hat{\boldsymbol{\lambda}}^E$. The chi-square statistic in equation (4.40) will be computed for testing the null hypothesis that the two SPDs are identical.

It is obvious that, with appropriate modifications, the above procedure can also be used to test the hypothesis that the risk-neutral measure $d\pi_I^*$ for physical investment returns is identical to the actual probability measure $d\pi$ over the states of the world. From equation (4.15), we know that

$$d\pi_I^* = d\pi \iff \tilde{\boldsymbol{\lambda}}^I = 0. \tag{4.42}$$

To test the hypothesis that all the λ^I's are equal to zero, we can apply a similar chi-square statistic as defined in (4.40) by replacing $\hat{\boldsymbol{\lambda}}^E$ with $\hat{\boldsymbol{\lambda}}^I$ and by using appropriate $\hat{H}_T$ and $\hat{B}_T$. In particular, $\hat{H}_T$ should be calculated by evaluating the Hessian matrix of $\hat{\Omega}(\boldsymbol{\lambda}^I)$ at $\hat{\boldsymbol{\lambda}}^I$, and $\hat{B}_T$ should be the covariance matrix of the gradient of $\exp[\sum_{i=1}^M \lambda_i^I (\tilde{R}_{it}^I - 1)]$ evaluated at $\hat{\boldsymbol{\lambda}}^I$.

5. DATA

I use the procedure proposed in chapter 4 to test the validity of the spanning assumption for the following six two-digit industries: mining, construction, manufacturing, transportation, communication, and utilities. Since the manufacturing industry may include subindustries with very different production characteristics, I further divide the manufacturing industry into two subgroups: more capital intensive manufacturing industry and less capital intensive manufacturing industry. Each industry is allowed to use a different production technology. The annual physical investment returns from 1949 to 1997 are constructed using equation (3.20). In the stock market, I formulate six industry portfolios, one for each industry listed above. The annual equity returns for the same time period are constructed as returns on investing in each industry portfolio. The remaining of this chapter provides a detailed description on the data sources, the criterion used to divide the manufacturing industry into two subgroups, and the formation of both physical investment returns and equity returns.

5.1 Data Sources

The annual physical capital investment data is provided by the Bureau of Economic Analysis (BEA) at the U.S. Department of Commerce. The BEA's primary data source is the annual plant and equipment survey, which provides investment data for non-residential investment by establishments engaged in non-farm industries. This data is supplemented by the quinquennial economic census and industry-specific data sources.

The BEA makes adjustments to the investment series using its judgment about the relative quality of relevant data sources so that the total annual investment across industries sums to the private investment in the National Income and Product Accounts. The BEA also provides the annual gross products and the year-end estimates of the capital stock at the industry level. All the nominal dollar values are deflated by the Producer Price Index of capital equipment provided by the Citibase.

The monthly returns on industry equity portfolios are downloaded from the NYSE, AMEX, and NASDAQ return files maintained by the Center for Research in Security Prices (CRSP). Risk free rates are from the Fama/Bliss Risk Free Rates File maintained by CRSP. Nominal returns are deflated using the Producer Price Index data from Citibase.

5.2 Classification of the Manufacturing Industry into Two Subgroups

The manufacturing industry consists of a wide range of industries with different production characteristics. According to the 1972-SIC basis, the 19 industries classified into the manufacturing family are broadly divided into two groups: industries producing durable goods and industries producing nondurable goods. Such classification is based on the different characteristics of final products rather than production technologies. Recall from chapter 3 that, consistent with Cobb-Douglas production functions, the technological difference across industries is characterized by the capital/labor ratio, i.e., the capital intensity. Since the reported capital stock series for each industry is not very reliable when compared with the investment series, I use the capital investment per worker as a proxy to capital intensity. Using the investment data from BEA and the number of employed persons reported in *Employment and Earnings*, I compute the capital investment per worker for each industry for the year 1997. Industries with invest-

ment per worker above (below) the median is classified as more (less) capital intensive manufacturing industry. The *more capital intensive manufacturing industry*(manfct1) includes rubber and miscellaneous plastics products, primary metal industries, stone, clay, and glass products, instruments and related products, electric and electronic equipment, tobacco products, paper and allied products, and petroleum and coal products. The *less capital intensive manufacturing industry*(manfct2) includes apparel and other textile products, leather and leather products, furniture and fixtures, printing and publishing, lumber and wood products, textile mill products, machinery, fabricated metal products, transportation equipment, and food and kindred products.

5.3 Formation of Equity Portfolio Returns

For each industry, I form value-weighted industry portfolios using all firms listed on NYSE, AMEX, and NASDAQ. The return files maintained by CRSP contain monthly return series for each industry portfolio.

One natural way to construct annual equity returns is to multiply the relevant monthly returns. However, this approach is not quite appropriate due to the different reporting frequencies for equity return data and physical capital investment data. Equity return is reported at high frequency, and is the instantaneous end of period value. However, physical investment data are reported, at best, as a quarterly average. The industry-level physical investment data are available only on an annual basis. Unlike equity returns, physical investment data are reported as the sum of capital investment expenditures over the period rather than the instantaneous end of period value. In other words, physical investment data are temporally aggregated. As pointed out by Porter (1999), such temporal aggregation induces (a) the reduction of the measured correlation between equity returns and physical investment returns, (b) a positive correlation between investment growth and equity returns lagged one period, and (c) a positive serial

Table 5.1   Summary Statistics

Industry	Parameters			Investment Return		Equity Return	
	α	η	δ	Mean	Std. Dev.	Mean	Std. Dev.
Mining	0.76	8.10	0.10	8.98 %	9.97 %	8.98 %	17.12 %
Constrn.	0.13	6.73	0.10	9.34 %	9.92 %	9.34 %	21.24 %
Manfct.	0.23	6.36	0.10	10.71 %	7.51 %	10.71 %	15.24 %
Manfct1.	0.20	7.26	0.10	10.55 %	7.67 %	10.55 %	16.18 %
Manfct2.	0.09	4.51	0.10	10.94 %	7.86 %	10.94 %	15.26 %
Transp.	0.76	18.03	0.10	8.78 %	10.08 %	8.78 %	18.15 %
Commun.	0.86	10.79	0.10	9.11 %	7.55 %	9.11 %	14.03 %
Utility	0.83	5.27	0.10	8.75 %	4.93 %	8.75 %	13.32 %

dependence in the first differences of physical investment returns. Porter (1999) compares several alternative methods for correcting temporal aggregation biases, and shows that using equity returns calculated from time averaged prices reduces the problems described above.

Following Porter (1999), I calculate annual returns on industry equity portfolios as follows:

$$R_j^E(t, t+1) = \frac{\sum_{n=1}^{12} P_{t+1,n}^j}{\sum_{n=1}^{12} P_{t,n}^j}, \qquad j = 1, 2, \ldots, N, \tag{5.1}$$

where t represents year, n represents month, j represents industry portfolio, and $P_{t,n}^j$ represents the equity price of portfolio j at month n in year t. Equity prices are constructed in an artificial way using the value-weighted industry portfolio returns including dividends. Clearly, equation (5.1) calculates the annual equity returns as the ratio of the average monthly price in year $t + 1$ over the average monthly price in year t.

Table 5.1 reports the mean and standard deviation of the estimated equity returns for each industry portfolio. The correlation matrix for all industry portfolios can be found in table 5.2.

Table 5.2   Correlation Matrix for Equity Returns

	Mining	Constrn.	Manufact.	Transp.	Commun.	Utility
Mining	1.00					
Constrn.	0.72	1.00				
Manufact.	0.58	0.67	1.00			
Transp.	0.53	0.62	0.83	1.00		
Commun.	0.10	0.35	0.62	0.63	1.00	
Utility	0.20	0.49	0.71	0.64	0.82	1.00

5.4   Estimation of Industry Physical Investment Returns

For each industry, I estimate the physical investment returns in three steps. First, I take the annual capital investment and gross product data for all of the six industries from BEA. Second, I arbitrarily set the depreciation rate $\delta = 0.10$, and construct time series of capital stock for each industry using the procedure stated below. Finally, I estimate the Cobb-Douglas coefficient α and the adjustment cost parameter η for each industry, and then use equation (3.20) to compute the estimated physical investment returns.

To construct the capital stock series, I start from equation (3.4), which characterizes the motion of the capital stock. Setting the value of capital stock at the beginning of 1948 equal to the reported value from BEA, I then construct the time series of capital stock for each industry according to equation (3.4) using the constant depreciation rate and the reported capital investment series from BEA.

Given the depreciation rate δ and the investment/capital ratios, the remaining two parameters to be determined are the Cobb-Douglas coefficient α and the adjustment cost parameter η. From equation (3.20) we know that α affects the mean of the physical investment return and η affects both the mean and the standard deviation. However, neither parameter has much impact on the correlation of the physical investment return with investment/capital ratios and with other variables. Following Cochrane (1991), I

choose α and η so that (a) the mean of physical investment returns is equal to the mean of equity returns and (b) the standard deviation of the fitted values of a regression of the physical investment returns on two leads and lags of the investment/capital ratio is equal to the standard deviation of the fitted value of the same regression for the equity returns. This choice of the standard deviation is designed to produce a physical investment return series of about the same standard deviation as the physical investment return component of equity returns. Since most of the empirical results are driven by the correlation of physical investment and equity returns, this scaling is not crucial to the results. Cochrane (1991) and Porter (1999) also point out that the correlation between equity returns and physical investment returns is mostly driven by the investment/capital ratio, and is not sensitive to the changes in parameter values.

With all the relevant information, I finally estimate the physical investment returns using equation (3.20). Table 5.1 reports the estimated parameter values and the mean and standard deviation of the estimated investment returns. The correlation matrix of physical investment returns for all industries is reported in table 5.3.

Table 5.3   Correlation Matrix for Physical Investment Returns

	Mining	Constrn.	Manufact.	Transp.	Commun.	Utility
Mining	1.00					
Constrn.	-0.06	1.00				
Manufact.	0.30	0.24	1.00			
Transp.	-0.08	0.59	0.32	1.00		
Commun.	0.06	0.38	0.52	0.41	1.00	
Utility	-0.20	0.40	0.13	0.46	0.28	1.00

6. EMPIRICAL RESULTS AND ROBUSTNESS CHECK

6.1 Empirical Results

With the constructed series for physical investment returns and equity returns, I apply the procedure proposed in chapter 4 to test whether the SPD recovered from physical investment returns can be used to price equity returns for the industries studied. The empirical results are reported in table 6.1. All panels of the table report the estimated values for the unknown parameters $\tilde{\lambda}^I$ and $\tilde{\lambda}^E$, the standard errors for the estimates, and the chi-square statistics for testing the null hypothesis (4.36) and (4.42). The estimated parameter vector $\hat{\lambda}^I$ identifies the state price probability density for physical investment returns, and is the numerical solution to the optimization problem (4.19). The estimated parameter vector $\hat{\lambda}^E$ identifies the state price probability density for equity returns, and is the numerical solution to the optimization problem (4.33). In both cases, the Quasi-Newton Method is used to find the numerical solutions. Standard errors for estimated Lagrange multipliers are reported in the parenthesis. Chi-square statistics reported in the last column are derived from (4.40). They are used to test the hypothesis that the risk-neutral probability measure pricing physical investment returns is identical to the actual probability measure, and the hypothesis that both physical investment market and financial market share the same risk-neutral probability measure.

Similar to all optimization problems, the Lagrange multipliers reflect the change in the value of the objective function as a result of a marginal change in the constraint set. In other words, the Lagrange multipliers are just the partial derivatives of the objective

Table 6.1 Parameter Estimates and Testing Statistics (I)

	Mining	Constrn	Manfct	Transp	Commun	Utility	χ^2
	Panel 1: including all six industries						
$\hat{\lambda}^I$	-2.47	0.84	-6.94	2.52	3.10	-22.53	25.76
	(2.22)	(2.92)	(4.03)	(2.61)	(3.81)	(5.63)	
$\hat{\lambda}^E$	1.75	-1.82	-6.73	5.03	-5.70	6.45	10.85
	(2.01)	(1.80)	(3.12)	(2.26)	(3.07)	(3.25)	
	Panel 2: excluding the manufacturing industry						
$\hat{\lambda}^I$	-4.46	-0.76		2.45	0.05	-22.71	25.21
	(1.98)	(2.71)		(2.62)	(2.93)	(5.34)	
$\hat{\lambda}^E$	-0.37	-1.36		2.45	-6.73	3.86	6.42
	(1.80)	(1.74)		(1.78)	(3.02)	(3.13)	
	Panel 3: excluding the utility industry						
$\hat{\lambda}^I$	-0.70	-0.97	-8.14	0.50	-0.42		14.93
	(1.91)	(2.46)	(3.25)	(2.25)	(3.34)		
$\hat{\lambda}^E$	1.04	-0.94	-4.23	3.95	-2.40		8.08
	(1.77)	(1.35)	(2.53)	(2.05)	(1.97)		
	Panel 4: excluding both the manufacturing and the utility industries						
$\hat{\lambda}^I$	-2.91	-2.04		-0.51	-4.11		11.74
	(1.80)	(2.29)		(2.22)	(2.61)		
$\hat{\lambda}^E$	0.07	-1.20		1.83	-3.08		4.94
	(1.64)	(1.26)		(1.60)	(1.81)		

function with respect to the constraints, and in this case are just marginal entropies. However, in the entropic framework, the Lagrange multipliers have more meaningful economic-statistical interpretation which can be summarized as follows: The λs reflect the "relative contribution" of each data point-constraint to the optimal objective value. Consequently, the λs reflect the information content of each constraint. According to the above interpretation, the magnitude of the estimated values of $\bar{\lambda}^I$ indicates the contribution of each industry's return series in identifying the SPD (or risk-neutral probability measure). Industries with the largest absolute values of λ^Is account for most of the deviation between the risk-neutral probability measure and the actual probability

measure. The opposite interpretation can be applied to industries with $\hat{\lambda}^I$s close to zero . Similarly, industries with the largest absolute values of $\hat{\lambda}^E$s account for most of the deviation between the risk-neutral probability measure in the physical investment market and that in the stock market. When the spanning assumption holds, all of the $\hat{\lambda}^E$s should equal to zero.

Panel 1 of table 6.1 presents the results when all six industries are included in the empirical testing. The utility industry and the manufacturing industry contribute most to identifying the SPD in the physical investment market with $\hat{\lambda}^I_{util} = -22.53$ and $\hat{\lambda}^I_{man} = -6.94$. The t-statistic for $\hat{\lambda}^I_{util}$ indicates that the Lagrange multiplier for the utility industry is significantly different from zero. According to the above interpretation, we conclude that the utility industry is the driving force for the deviation of the risk-neutral probability measure in the physical investment market from the actual probability measure. Furthermore, the chi-square statistic for testing the joint hypothesis that all of the Lagrange multipliers are equal to zero yields a value of 25.76. Since the chi-square statistic is significantly higher than the 5% critical value, I reject the null hypothesis that the risk-neutral probability measure is equal to the actual probability measure in the physical investment market.

The last two rows of panel 1 report the parameter estimates and testing statistics for the equity market using the SPD recovered from the physical investment market as the prior. The Lagrange multipliers $\hat{\lambda}^E_{man} = -6.73$, $\hat{\lambda}^E_{tran} = 5.03$, and $\hat{\lambda}^E_{util} = 6.45$ are significantly different from zero at the 5 percent significance level, indicating that the manufacturing, the transportation, and the utility industries contribute most to the deviation of the risk-neutral probability measure in the equity market from that recovered from the physical investment returns. However, the chi-square statistic testing the joint hypothesis that all of the Lagrange multipliers are equal to zero yields a value of 10.85, which is less than the 5% critical value. Hence, I can not reject the hypothesis that the risk-neutral measure in the equity market is identical to that in the physical

investment market. In other words, this provides supporting evidence that the payoff space of physical investment returns spans that of equity returns. Therefore, when all six industries are included, the physical investment returns contain sufficient information to correctly price the corresponding equity returns.

To examine whether the empirical results are sensitive to the industries included in the study, I apply the same procedure to cases in which one or two industries are excluded from the estimation and testing. Panels 2 and 3 report parameter estimates and testing statistics when either the manufacturing industry or the utility industry is excluded. while panel 4 presents the results when both the manufacturing and the utility industry are excluded from estimation.

As shown in panel 2, the utility industry is again the driving force for identifying the SPD in the physical investment market with $\hat{\lambda}^I_{util} = -22.71$ and a highly significant t-statistic. The estimated Lagrange multiplier for the mining industry ($\hat{\lambda}^I_{mine} = -4.46$) is also significantly different from zero. The chi-square statistic for testing the joint hypothesis that all the Lagrange multipliers are equal to zero takes a value of 25.21, leading to the rejection that the risk-neutral measure recovered from the physical investment returns is identical to the actual probability measure. Using the SPD for the physical investment market as the prior, I further estimate the corresponding SPD embedded in the equity portfolio returns. The results show that the communication industry contributes most to the deviation of risk-neutral measures between physical investment returns and equity returns. The estimated Lagrange multiplier for the communication industry is -6.73. and is significantly different from zero. The chi-square statistic testing the hypothesis that both the physical investment returns and the equity returns share the same risk-neutral measure yields a value of 6.42, which is lower than the 5% critical value with 5 degrees of freedom. Again, the empirical test without the manufacturing industry can not reject the hypothesis that the payoff space of physical investment returns spans that of equity returns.

Panel 3 reports the estimation results when the utility industry is excluded from the study. Similar to the previous results, the risk-neutral probability measure recovered from the physical investment returns is significantly different from the actual probability measure (chi-square statistic=14.93). The estimation results using the equity returns (see the last two rows of panel 3) suggest that the SPD for the physical investment can be used to correctly price the equity returns. None of estimated Lagrange multipliers is significantly different from zero. The joint significance test yields a chi-square statistic of 8.08. smaller than the 5% critical value with 5 degrees of freedom. Hence, when the utility industry is excluded, we still can not reject the spanning assumption.

Panel 4 recovers and compares the SPDs using only four industries: mining, construction, transportation, and communication. The results indicate that the communication industry contributes the most to the deviation between the two risk-neutral measures. The estimated Lagrange multiplier is -3.08, but is not significant at the 5% level. Further. the chi-square statistic from the joint significance test only takes a value of 4.94, far below the critical value with four degrees of freedom. Again. I find supporting evidence that the payoff space of the physical investment spans that of financial securities.

Finally, I address the concern that the manufacturing industry may include subindustries with very different production characteristics. Thus, it may not be appropriate to group them together and use one set of coefficients (α, δ, and η) to represent their production technologies. For the 19 two-digit industries grouped into the manufacturing family, I further divide them into two subgroups: more capital intensive manufacturing industry (manfct1) and less capital intensive manufacturing industry (manfct2). Table 6.2 presents the estimation results when the finer industry classification described above is used. Among the seven industries included in the study, the manfct2 and the utility industries are the most important factors "driving" the risk-neutral measure for the physical investment away from the actual probability measure. The estimated Lagrange multipliers for these two industries are $\hat{\lambda}'_{man2} = -14.21$ and $\hat{\lambda}'_{util} = -26.56$.

Table 6.2 Parameter Estimates and Testing Statistics (II)

	Mining	Constrn	Manfct1	Manfct2	Transp	Commun	Utility	$\chi^2_{(7)}$
$\hat{\lambda}^I$	-3.69	-1.56	9.10	-14.21	3.91	2.20	-26.56	27.54
	(2.38)	(3.39)	(5.37)	(5.21)	(2.83)	(4.28)	(6.38)	
$\hat{\lambda}^E$	2.60	-2.60	0.82	-7.17	3.79	-5.76	6.98	12.89
	(2.21)	(1.99)	(5.33)	(3.72)	(2.62)	(3.47)	(3.29)	

respectively. Both of them are significantly different from zero. The large chi-square statistic (27.54) measuring the distance between the two probability measures indicates that the risk-neutral measure recovered from the physical investment returns is significantly different from the actual probability measure characterizing the states of the world.

As before, the last two rows of the table present the estimated Lagrange multipliers identifying the SPD for the equity market. Clearly, the set of estimates as a whole is not significantly different from zero. The joint significance test yields a chi-square statistic of 12.89, which is below the 5% critical value with 7 degrees of freedom. Hence, again, I find supporting evidence that the payoff space of the physical investment spans that of financial securities.

In summary, I apply the proposed method to test the spanning assumption using return series from mining, construction, manufacturing, communication, transportation, and public utility industries. Empirical results show that the physical investment returns are closely related to the equity returns at the cross-industry level in the sense that the SPD (risk-neutral measure) recovered from the physical investment returns is able to correctly (in a statistical sense) price the equity returns. This indicates that physical capital investment conveys crucial, if not exclusive, information on financial asset pricing. The intuition behind this result is as follows. The fundamental source of uncertainty in the stock market is the business cycle induced by the real macroeconomic risks.

Facing a productivity shock, firms are forced to alter their intertemporal production and capital investment plans accordingly. Moreover, firms with different production technologies will react quite differently in response to the same economy wide shock. Such heterogeneous reaction to macroeconomic risks is the fundamental reason for both the time-series and cross-sectional variations in physical investment returns. If we view the financial securities traded on NYSE as claims to different combinations of all the production technologies in the economy, then we should expect that the variations in both physical investment returns and equity returns are driven by the same set of real factors. Hence, we should be able to infer all the information necessary for pricing financial assets from firms' physical capital investment decisions.

One immediate implication of the above results is that any asset pricing model should at least capture the pricing information embedded in physical capital investment in order to generate successful empirical results. The model should either explicitly incorporate the presence of macroeconomic risks affecting firms' physical investment decisions or use appropriate proxies to capture such effect. This may provide an alternative explanation for the disappointing empirical performance of the CAPM and the CCAPM. In the tradition CAPM, market return is used as the only factor explaining the variations in equity returns. Its empirical failure may be due to the fact that market return alone is not able to capture all the important intertemporal investment opportunities in the economy. The CCAPM attempts to infer the effects of macroeconomic risks on equity returns through the changes in consumption decisions. However, empirical studies have shown that consumption change is a bad proxy for the effects of macroeconomic risks since nondurable consumption growth barely moves over the business cycle.

Another implication of the empirical results is related to the importance of developing asset pricing models that incorporate key production characteristics. The finance literature has been dominated by asset pricing models focusing exclusively on the financial sector of the economy. These models attempt to explain the expected return

of a particular financial asset by its covariance with other assets' returns. Numerous empirical works in this area have not generated satisfactory empirical results. On the other hand, relatively little effort has been made to explicitly model the pricing impacts of key production characteristics, e.g., the cost of adjusting capital stock. As mentioned before, the fundamental source of uncertainty in the stock market is the business cycle induced by the macroeconomic risks. Moreover, the reaction of the firm's physical capital investment decision in response to macroeconomic risks is determined by its production characteristics. Since empirical results show that physical capital investment conveys crucial information necessary for pricing financial securities, key production characteristics must have a nontrivial impact on financial asset pricing. I believe that models explicitly incorporating key production characteristics will generate much richer testable implications than models focusing exclusively on the financial market.

6.2 Robustness Check

In this section, I perform robustness check on the empirical results derived in the last section. In particular, I examine whether the results are sensitive to reasonable changes of parameter values (α, η and δ) and of the functional form of adjustment costs.

6.2.1 Other Specifications of Parameter Values

To check the robustness of specification with respect to production parameters α, η and δ, I change the parameter values within reasonable ranges around the estimates reported in table 5.1. As before, I assume that all six industries have the same depreciation rate. For each value of δ from 0.06 to 0.18 (with an increment of 0.03 each time), I estimate α and η for each industry to equate the mean physical investment return to the mean equity return and to equate the standard deviation of the fitted values of a regression of the physical investment returns on two leads and lags of the investment/capital

Table 6.3   Different Specifications of Production Parameters

Industry	$\delta = 0.06$		$\delta = 0.09$		$\delta = 0.12$		$\delta = 0.15$		$\delta = 0.18$	
	α	η	α	η	α	η	α	η	α	η
Mining	0.82	10.20	0.79	9.06	0.73	7.14	0.68	5.70	0.64	4.65
Constrn.	0.17	11.46	0.14	7.63	0.12	5.50	0.11	4.36	0.10	3.24
Manfct.	0.24	6.43	0.23	6.30	0.22	5.62	0.20	4.20	0.19	3.57
Transp.	0.83	21.20	0.78	19.29	0.75	17.63	0.76	17.45	0.70	14.28
Commun.	0.87	12.23	0.86	11.03	0.85	10.19	0.83	9.07	0.81	8.07
Utility	0.87	5.88	0.84	5.33	0.84	5.39	0.85	5.50	0.83	5.08

ratio to the standard deviation of the fitted value of the same regression for the equity returns. The estimated parameter values are reported in table 6.3. As one can see, for all the six industries, the parameter values vary within reasonable ranges around the estimates reported in table 5.1.

For each set of new parameter estimates, I construct the physical investment return series using equation (3.20). Then I apply the procedure proposed in chapter 4 to examine the validity of the spanning assumption. The parameter estimates and testing statistics are reported in table 6.4. The estimated parameter vector $\hat{\lambda}^I$ identifies the state price probability density for physical investment returns, and is the numerical solution to the optimization problem (4.19). The estimated parameter vector $\hat{\lambda}^E$ identifies the state price probability density for equity returns, and is the numerical solution to the optimization problem (4.33). In both cases, the Quasi-Newton Method is used to find the numerical solutions. Standard errors for estimated Lagrange multipliers are reported in the parenthesis. Chi-square statistics reported in the last column are derived from (4.40). They are used to test the hypothesis that the risk-neutral probability measure pricing physical investment returns is identical to the actual probability measure and the hypothesis that both physical investment market and financial market share the same risk-neutral probability measure.

Table 6.4   Robustness Check (I)

	Mining	Constrn.	Manfct.	Transp.	Commun.	Utility	$\chi^2_{(6)}$
	Panel 1: $\delta = 0.06$						
$\hat{\lambda}^I$	-2.33	1.46	-8.62	2.33	4.28	-26.15	26.93
	(2.44)	(3.15)	(5.27)	(2.96)	(4.47)	(6.62)	
$\hat{\lambda}^E$	2.02	-1.93	-6.89	4.66	-6.42	8.11	12.53
	(2.03)	(1.85)	(3.04)	(2.30)	(3.09)	(3.31)	
	Panel 2: $\delta = 0.09$						
$\hat{\lambda}^I$	-2.31	1.11	-7.73	2.57	3.58	-24.04	26.44
	(2.23)	(3.01)	(4.37)	(2.67)	(4.01)	(5.95)	
$\hat{\lambda}^E$	1.83	-1.88	-6.73	4.94	-5.96	6.94	11.14
	(2.03)	(1.83)	(3.14)	(2.29)	(3.11)	(3.28)	
	Panel 3: $\delta = 0.12$						
$\hat{\lambda}^I$	-2.27	0.30	-6.86	2.51	2.59	-18.50	23.97
	(2.13)	(2.75)	(3.69)	(2.44)	(3.57)	(4.30)	
$\hat{\lambda}^E$	1.64	-1.70	-6.73	5.17	-5.23	5.57	10.57
	(1.95)	(1.74)	(3.03)	(2.22)	(3.00)	(3.17)	
	Panel 4: $\delta = 0.15$						
$\hat{\lambda}^I$	-1.90	-0.25	-7.54	2.52	2.41	-14.00	21.95
	(2.06)	(2.53)	(3.52)	(2.24)	(3.39)	(4.18)	
$\hat{\lambda}^E$	1.53	-1.56	-6.71	5.26	-4.75	4.65	10.44
	(1.89)	(1.67)	(2.91)	(2.20)	(2.95)	(3.08)	
	Panel 5: $\delta = 0.18$						
$\hat{\lambda}^I$	-2.00	-0.73	-6.86	2.54	2.21	-12.11	21.09
	(2.02)	(2.43)	(3.28)	(2.19)	(3.23)	(3.78)	
$\hat{\lambda}^E$	1.38	-1.43	-6.69	5.30	-4.22	3.91	10.32
	(1.87)	(1.62)	(2.86)	(2.16)	(2.90)	(3.03)	

Panel 4 of table 6.4 presents the results when all six industries are assumed to have the same depreciation rate $\delta = 0.15$. Clearly, the qualitative results are very similar to those reported in table 6.1. The utility industry and the manufacturing industry contribute most to identifying the SPD in the physical investment market with $\hat{\lambda}^I_{util} = -14.00$ and $\hat{\lambda}^I_{man} = -7.54$. The t-statistics indicate that both Lagrange multipliers are significantly different from zero. According to the previous interpretation, I conclude that

the utility and the manufacturing industries are the driving force for the deviation of the risk-neutral probability measure in the physical investment market from the actual probability measure. Furthermore, the chi-square statistic for testing the joint hypothesis that all the Lagrange multipliers are equal to zero yields a value of 21.95. Since the chi-square statistic is significantly higher than the 5% critical value, we reject the null hypothesis that the risk-neutral probability measure is identical to the actual probability measure in the physical investment market.

The last two rows of panel 4 report the parameter estimates and testing statistics for the equity market using the SPD recovered from the physical investment market as the prior. The Lagrange multipliers $\hat{\lambda}^E_{man} = -6.71$ and $\hat{\lambda}^E_{tran} = 5.26$ are significantly different from zero at the 5 percent significance level, indicating that the manufacturing and the transportation industries contribute most to the deviation of the risk-neutral probability measure in the equity market from that recovered from the physical investment returns. However, the chi-square statistic testing the joint hypothesis that all the Lagrange multipliers are equal to zero yields a value of 10.44, which is less than the 5% critical value. Hence, I can not reject the hypothesis that the risk-neutral measure in the equity market is identical to that in the physical investment market.

Panels 1 through 3 and panel 5 report qualitatively similar results. The chi-square statistics for testing the spanning assumption are 12.53, 11.14, 10.57, and 10.32, respectively. None of them is significant at 5% significance level. In other words, these provide supporting evidence that the payoff space of the physical investment returns spans the payoff space of the equity returns.

In summary, the empirical results derived in section 6.1 are quite robust with respect to different specifications of the depreciation rates, the Cobb-Douglas coefficients, and the adjustment cost coefficients.

6.2.2 Asymmetric Adjustment Cost Function

Now I turn to a different specification of the adjustment cost function. In chapter 3, I adopt a symmetric convex (quadratic) function to capture the costs of adjusting capital stocks. However, as pointed out by many authors, there is no reason to believe that the cost of positive adjustment in the capital stock would be the same as that of an equal-size negative adjustment. Following Hamermesh and Pfann (1996), I consider the following convex adjustment cost function that allows for asymmetry in marginal costs and contains equation (3.16) as a special case:

$$A(I_t, K_t) = \{\exp[\eta_1(\frac{I_t}{K_t} - \eta_0)] - \eta_1(\frac{I_t}{K_t} - \eta_0) + \frac{\eta_2}{2}(\frac{I_t}{K_t} - \eta_0)^2 - 1\}K_t, \qquad (6.1)$$

where η_0 is the rate of physical capital investment that entails no adjustment costs, η_1 and $\eta_2 \geq 0$ denote other parameters, and $\exp(\cdot)$ is the exponential function. Clearly, adjustment cost function (6.1) is linear homogeneous in I_t and K_t, and thus is consistent with the basic assumption made by the literature on the q-theory of investment. Note that equation (6.1) reduces to the symmetric adjustment cost function (3.16) when $\eta_1 = 0$ and $\eta_0 = 0$. However, when $\eta_1 \neq 0$, equation (6.1) allows for asymmetric adjustment costs. If $\eta_1 > (<)0$, a rate of investment I_t/K_t higher than η_0 entails greater (smaller) adjustment costs than an equal-size downward adjustment. Taking derivatives with respect to I_t and K_t gives

$$A_I(t) = \eta_1 \exp[\eta_1(\frac{I_t}{K_t} - \eta_0)] - \eta_1 + \eta_2(\frac{I_t}{K_t} - \eta_0), \qquad (6.2)$$

$$A_K(t) = (1 - \eta_1\frac{I_t}{K_t})\exp[\eta_1(\frac{I_t}{K_t} - \eta_0)] + \eta_1\eta_0 - \frac{1}{2}\eta_2(\frac{I_t}{K_t} - \eta_0)(\frac{I_t}{K_t} + \eta_0) - 1. (6.3)$$

Hence, the physical investment return is defined by equation (3.14) in which $\epsilon_t F_K(t)$, $A_I(t)$, and $A_K(t)$ are given by equation (3.17), (6.2) and (6.3), respectively.

For estimation purposes, I set $\delta = 0.10$ and $\eta_0 = \delta$ for all industries. This implies that no adjustment costs will occur if physical investment is made just to compensate for the capital loss due to depreciation. For simplicity, I further assume that η_1 is positive and

constant across all industries. With a positive value of η_1, I essentially assume that the adjustment costs of an upward adjustment exceed the adjustment costs of an equal-size downward adjustment. To ease the computation burden, I allow η_1 to take values from 0.1 to 0.5 (with an increment of 0.1 each time). For 100% increase of the capital stock (i.e., $I_t/K_t = 1$) with no depreciation and $\eta_2 = 0$, the above values of η_1 correspond to adjustment costs of 0.52%, 2.14%, 4.98%, 9.18%, and 14.8% of the current capital stock, respectively. Given the values of δ and η_1, I apply the procedure proposed in section 5.4 to estimate the other unknown parameters (α and η_2) in the physical investment return formula. The three panels in table 6.5 report the estimated parameter values for $\eta_1 = 0.10$, $\eta_1 = 0.30$, and $\eta_1 = 0.50$, respectively. Clearly, the parameter estimates are not very sensitive to the change of η_1. With these parameter estimates, I then construct the physical investment return series according to equation (3.14).

Table 6.5   Parameter Estimates with Asymmetric Adjustment Costs

	Mining	Constrn	Manfct	Transp	Commun	Utility
	Panel 1					
α	0.48	0.09	0.15	0.34	0.49	0.60
η_0	0.10	0.10	0.10	0.10	0.10	0.10
η_1	0.10	0.10	0.10	0.10	0.10	0.10
η_2	4.70	4.20	3.40	6.38	8.11	5.70
δ	0.10	0.10	0.10	0.10	0.10	0.10
	Panel 2					
α	0.48	0.09	0.15	0.34	0.49	0.60
η_0	0.10	0.10	0.10	0.10	0.10	0.10
η_1	0.30	0.30	0.30	0.30	0.30	0.30
η_2	4.60	4.10	3.30	6.30	8.30	5.60
δ	0.10	0.10	0.10	0.10	0.10	0.10
	Panel 3					
α	0.48	0.09	0.15	0.34	0.48	0.60
η_0	0.10	0.10	0.10	0.10	0.10	0.10
η_1	0.50	0.50	0.50	0.50	0.50	0.50
η_2	4.50	4.17	3.10	6.70	7.47	5.40
δ	0.10	0.10	0.10	0.10	0.10	0.10

With the newly constructed physical investment returns, I examine the validity of the spanning assumption by using the nonparametric procedure proposed in chapter 4. Panel 1 of table 6.6 reports the parameter estimates and testing statistics when $\eta_1 = 0.10$. Similar to the previous results, the manufacturing industry and the utility industry are the driving force for identifying the SPD in the physical investment market with $\hat{\lambda}^I_{man} = -9.68$ and $\hat{\lambda}^I_{util} = -7.28$ and highly significant t-statistics. The chi-square statistic for testing the hypothesis that the risk-neutral measure recovered from the physical investment returns is identical to the actual probability measure takes a value of 18.94, leading to the rejection of the above hypothesis. However, the chi-square statistic measuring the deviation of the risk-neutral measure for the equity returns from the risk-neutral measure for the physical investment returns yields a much smaller value of 9.52, less than the 5% critical value with 6 degrees of freedom. This implies that one can not reject the hypothesis that the two risk-neutral measures are identical.

Table 6.6   Robustness Check (II)

	Mining	Constrn.	Manfct.	Transp.	Commun.	Utility	$\chi^2_{(6)}$
	Panel 1: $\eta_1 = 0.10$						
$\hat{\lambda}^I$	-0.84	0.26	-9.68	0.44	2.58	-7.28	18.94
	(1.90)	(2.50)	(3.69)	(2.25)	(2.56)	(3.05)	
$\hat{\lambda}^E$	1.28	-1.34	-5.75	4.65	-4.47	3.55	9.52
	(1.78)	(1.57)	(2.66)	(2.13)	(2.80)	(2.98)	
	Panel 2: $\eta_1 = 0.30$						
$\hat{\lambda}^I$	-0.83	0.25	-9.83	0.45	2.65	-7.33	19.07
	(1.90)	(2.51)	(3.71)	(2.25)	(2.50)	(3.07)	
$\hat{\lambda}^E$	1.27	-1.33	-5.75	4.66	-4.48	3.56	9.51
	(1.78)	(1.57)	(2.66)	(2.14)	(2.80)	(2.98)	
	Panel 3: $\eta_1 = 0.50$						
$\hat{\lambda}^I$	-0.76	0.22	-10.10	0.82	2.58	-7.52	19.36
	(1.89)	(2.42)	(3.75)	(2.11)	(2.68)	(3.12)	
$\hat{\lambda}^E$	1.32	-1.19	-5.88	4.70	-4.69	3.86	9.80
	(1.79)	(1.58)	(2.67)	(2.15)	(2.82)	(2.98)	

The last two panels of table 6.6 present estimation and testing results for $\eta_1 = 0.30$ and $\eta_1 = 0.50$, respectively. The chi-square statistics testing the validity of the spanning assumption are 9.51 and 9.80. Neither of them are significant at the 5% level. Again, I find supporting evidence for the spanning assumption. Hence, the previous empirical results are not sensitive to different specifications of the adjustment cost function.

7. TESTING THE PHYSICAL INVESTMENT FACTOR PRICING MODEL

In the previous chapters, I propose a state price density approach to examining the validity of the spanning assumption. Empirical tests using the cross-industry data provide supporting evidence for the hypothesis that the payoff space of physical investment spans the payoff space of financial investment. As argued in chapter 4, if the law of one price holds, the spanning assumption immediately implies that there exists a stochastic discount factor m that is a linear combination of the physical investment returns and that correctly prices all equity returns. In this chapter, I study the performance of such a linear factor pricing model within the GMM framework. In particular, I focus on whether the testing results are sensitive to different aggregations of physical investment returns and how the state price density approach is related to the traditional linear factor pricing approach.

7.1 Estimation Method

Following Cochrane (1996), I test the conditional predictions of the following asset pricing model:

$$E(m_{t+1}R_{t+1}^E \mid I_t) = 1, \tag{7.1}$$

where

$$m_{t+1} = b_0 + \sum_{i=1}^{M} b_i R_{i,t+1}^I. \tag{7.2}$$

Here, the t subscript denotes time, R^E is an $N \times 1$ vector of equity portfolio returns, m_{t+1} is the stochastic discount factor expressed as a linear combination of M physical investment returns $R^I_{i,t+1}$ ($i = 1, 2, \ldots, M$), and I_t denotes the information set containing all the information available at time t.

For notation purposes, define

$$h(R^E_{t+1}, R^I_{t+1}, b) \equiv m_{t+1} R^E_{t+1} - 1, \tag{7.3}$$

where R^I_{t+1} is an $M \times 1$ vector of physical investment returns, and b is an $(M + 1) \times 1$ vector of factor loadings. Hence, equation (7.1) can be written as

$$E[h(R^E_{t+1}, R^I_{t+1}, b) \mid I_t] = 0. \tag{7.4}$$

Let z_t be a q-dimensional vector of variables that is observable at time t, $z_t \in I_t$. Using an iterated expectation argument, I can derive the following alternative expression for the restrictions in equation (7.1):

$$E[h(R^E_{t+1}, R^I_{t+1}, b) \otimes z_t] = 0, \tag{7.5}$$

where $\otimes$ denotes the Kronecker product. It is obvious that the conditional restriction (7.4) implies the unconditional restriction (7.5). Conversely, if (7.5) holds for all the instruments z_t in the information set I_t, then equation (7.4) holds. Hence, one can test all the implications of (7.4) by testing the unconditional restriction (7.5), which is easier to implement. Of course, it is impossible to identify and include all of the relevant instruments in the empirical tests. In general, one only uses a few carefully chosen variables which are most relevant.

The conditions in equation (7.5) are known as the population orthogonality conditions. I will use these conditions to derive a consistent estimator of the unknown parameter vector b. Let $f(R^E_{t+1}, R^I_{t+1}, z_t, b)$ be an $Nq \times 1$ vector such that

$$f(R^E_{t+1}, R^I_{t+1}, z_t, b) \equiv h(R^E_{t+1}, R^I_{t+1}, b) \otimes z_t. \tag{7.6}$$

Then the orthogonality conditions can be written as

$$E[f(\mathbf{R}^E_{t+1}, \mathbf{R}^I_{t+1}, \mathbf{z}_t, \mathbf{b})] = \mathbf{0}. \tag{7.7}$$

The sample counterpart of the left-hand side term in (7.7) is defined as

$$g_T(\mathbf{b}) \equiv \frac{1}{T} \sum_{t=1}^{T} f(\mathbf{R}^E_{t+1}, \mathbf{R}^I_{t+1}, \mathbf{z}_t, \mathbf{b}), \tag{7.8}$$

where T is the length of the sample period. I assume that a law of large numbers can be applied to $g_T(\mathbf{b})$ so that it converges to its population mean for all $\mathbf{b}$ with probability one:

$$\lim_{T \to \infty} g_T(\mathbf{b}) = E[f(\mathbf{R}^E_{t+1}, \mathbf{R}^I_{t+1}, \mathbf{z}_t, \mathbf{b})], \tag{7.9}$$

almost surely.

Following Hansen's GMM approach, I estimate the unknown parameter vector $\mathbf{b}$ by minimizing the following quadratic form:

$$J_T(\mathbf{b}) = g_T(\mathbf{b})' W_T g_T(\mathbf{b}) \tag{7.10}$$

with respect to $\mathbf{b}$. Here, W_T is a positive definite weighting matrix which converges in probability to a positive definite matrix W_0.

Under some regularity conditions, the GMM estimator $\mathbf{b}_T$ is a consistent estimator of $\mathbf{b}$. Furthermore, one can apply a central limit theorem to show that $\mathbf{b}_T$ is asymptotically normally distributed. Define a sequence of Nq-dimensional random vectors as $f_{t+1} \equiv f(\mathbf{R}^E_{t+1}, \mathbf{R}^I_{t+1}, \mathbf{z}_t, \mathbf{b})$ and a covariance matrix $S_f \equiv \lim_{j \to \infty} \sum_{-j}^{j} E(f_{t+1} f'_{t+1-j})$. Then $\sqrt{T}(\mathbf{b}_T - \mathbf{b})$ is asymptotically normally distributed with mean zero and covariance matrix

$$\Lambda = (D_0' W_0 D_0)^{-1} D_0' W_0 S_f W_0 D_0 (D_0' W_0 D_0)^{-1'}, \tag{7.11}$$

where

$$D_0 = E \left[\frac{\partial f(\mathbf{R}^E_{t+1}, \mathbf{R}^I_{t+1}, \mathbf{z}_t, \mathbf{b})}{\partial \mathbf{b}} \right]. \tag{7.12}$$

To obtain the asymptotic efficient GMM estimator, one needs to set $W_0 = S_f^{-1}$. In this case, the asymptotic covariance matrix is reduced to

$$\Lambda = (D_0' S_f^{-1} D_0)^{-1}. \tag{7.13}$$

To empirically estimate the unknown parameter vector b in the physical investment factor pricing model, I follow the commonly used 2-stage GMM algorithm (see Altug and Labadie (1994)).

1. Minimize $g_T(b)' g_T(b)$ with respect to b. In the first stage, one essentially minimizes $J_T(b)$ by setting $W_T = I$. For notation purposes, denote the first-stage estimator by $\hat{b}_T$.

2. Estimate f_{t+1} by setting $\hat{f}_{t+1} = h(R_{t+1}^E, R_{t+1}^I, \hat{b}_T) \otimes z_t$ and use the estimated residuals to form a consistent estimator of S_f, denoted by S_T.

3. Minimize $g_T(b)' S_T^{-1} g_T(b)$ with respect to b. The resulting second-stage estimator b_T is asymptotically efficient with standard errors given by equation (7.13).

To empirically test the validity of the physical investment factor pricing model, I essentially need to test the null hypothesis that all the orthogonality conditions in equation (7.5) hold. The proposed testing statistic is defined as T times the minimized value of the objective function (7.10):

$$T J_T(b_T) \equiv T g_T(b_T)' S_T^{-1} g_T(b_T). \tag{7.14}$$

As an extension of the specification test in Sargan(1958) and Ferguson(1958), Hansen(1982) shows that

$$T J_T(b_T) \rightarrow \chi_{Nq-M-1}^2, \tag{7.15}$$

where χ_{Nq-M-1}^2 is a chi-square random variable with $(Nq - M - 1)$ degrees of freedom. If all of the orthogonality conditions are satisfied, then the sample estimate of the test statistic $T J_T$ should be close to zero. The above test is called the J_T test in the literature. The chi-square statistic is usually referred to as the J_T statistic.

7.2 Testing the Physical Investment Factor Pricing Model

To examine the validity of the physical investment factor pricing model, I use the same equity return series used in the entropic analysis. For each industry, I use the physical investment return series constructed by using the parameter estimates reported in table 5.1. To be consistent with the PCAPM, I use the capital-weighted average of industry physical investment returns as the instrumental variable. Such an instrument is selected because the PCAPM suggests that firms' physical capital investment is the linkage between macroeconomic risks and equity returns. In addition to the theoretical prediction, empirical results in chapter 6 also provide strong supporting evidence that physical investment returns contain sufficient information that can be used to correctly price equity returns. Hence, the weighted average of physical investment returns is a pivotal variable in forecasting equity returns, and thus is an appropriate instrument for the GMM estimation. Following Cochrane (1996) and Kasa (1997), the instrument is lagged twice to avoid overlapping with the equity return series. The six industry no-arbitrage constraints plus the common instrument for each industry result in a system of 12 orthogonality (moment) conditions.

One may want to argue that I should incorporate a more extensive list of instrumental variables. It is true that good instruments can enhance the power and reliability of the GMM results. Hansen (1985) provides some discussions on the optimal selection of instruments. However, the proposed methods tend to be difficult for empirical implementation. In empirical applications the instrument selection relies most on model prediction, previous empirical evidence, and subjective judgment. Without knowing the true list of all the relevant instruments, I adopt the conservative approach of only including the instrument that is supported by both the theoretical model and the empirical evidence. One other reason that I do not consider a more extensive set of instruments is related to the relatively small size of my sample. Kocherlakota (1990) presents sim-

ulation results indicating that the small sample properties of the GMM estimates and test statistics deteriorate as the number of instruments increases.

7.2.1 Different Aggregations of Physical Investment Factors

Previous research has used one or two aggregate production technologies and the corresponding physical investment returns as factors to explain the expected equity returns. To examine whether the empirical results are sensitive to different aggregations of physical investment factors, I use the GMM method to estimate and test four factor pricing models with different levels of aggregation for the physical investment factors.

The first linear factor pricing model (Model 1) uses the capital-weighted average of industry physical investment returns as the only factor, and is specified as follows:

$$m_{t+1} = b_0 + b_1 R^I_{avg,t+1},$$ (7.16)

where

$$R^I_{avg,t+1} \equiv \frac{1}{\sum_{i=1}^{6} K_{it+1}} \sum_{i=1}^{6} K_{it+1} R^I_{it+1}.$$ (7.17)

Here, R^I_{it+1} and K^I_{it+1} denote the physical investment return and the physical capital stock for industry i at time $t+1$. The above model essentially aggregates the six industry physical investment returns into one single factor. The aggregated investment return $R^I_{avg,t+1}$ can also be interpreted as the return generated by one aggregate production technology.

The second factor pricing model (Model 2) uses two factors to construct the stochastic discount factor m. One is the capital-weighted average of physical investment returns for the mining, transportation, communication, and utility industries, denoted by $R^I_{avg1,t+1}$. The other is the capital-weighted average of physical investment returns for the construction and manufacturing industries, denoted by $R^I_{avg2,t+1}$. The model can then be written as

$$m_{t+1} = b_0 + b_1 R^I_{avg1,t+1} + b_2 R^I_{avg2,t+1}.$$ (7.18)

Here, I aggregate the physical investment returns of the six industries into two factors. The first factor represents industries which are more capital intensive. The estimated αs (see table 5.1) for the four industries aggregated into the first factor are all above 0.75. The second factor represents industries which are less capital intensive, since the estimated αs for the construction and manufacturing industries are below 0.25. Therefore, the two factors in equation (7.18) may also be interpreted as the physical investment returns generated by two aggregate production technologies with different capital intensity.

The third factor pricing model (Model 3) studied uses three factors to explain the cross-industry variation in equity returns. The model can be stated as

$$m_{t+1} = b_0 + b_1 R^I_{man,t+1} + b_2 R^I_{util,t+1} + b_3 R^I_{avg3,t+1}, \tag{7.19}$$

where $R^I_{man,t+1}$ denotes the physical investment return for the manufacturing industry, $R^I_{util,t+1}$ denotes the physical investment return for the utility industry, and $R^I_{avg3,t+1}$ denotes the capital-weighted average of physical investment returns for the mining, construction, transportation, and communication industries. Since the entropic results in chapter 6 indicate that the manufacturing industry and the utility industry contribute most for identifying the SPD in physical investment market, the above model incorporates the physical investment returns for these two industries as two separate factors. Similar to the first two models, the weighted average physical investment return for the remaining four industries may be interpreted as a return generated by one aggregate production technology.

The fourth and final linear factor pricing model (Model 4) examined uses all six physical investment returns as separate factors. Specifically,

$$m_{t+1} = b_0 + \sum_{i=1}^{6} b_i R^I_{i,t+1}. \tag{7.20}$$

Clearly, the model allows for the physical investment return of each industry to be generated by a separate production technology.

In summary, the first two factor pricing models use one or two highly aggregated factors to construct the stochastic discount factor m. This is the traditional approach adopted by Cochrane (1996) and Kasa (1997) in the empirical PCAPM literature. The last two factor pricing models use relatively disaggregated production technologies to construct factors. Such an approach may alleviate the potential joint hypothesis test problem encountered by the traditional approach. For the remaining of the section, I apply the GMM method to estimate and test all four factor pricing models, and pay special attention to whether the empirical results are sensitive to different aggregations of the physical investment factors.

7.2.2 Estimation Results

Table 7.1 reports the GMM estimation results for the above four physical investment factor pricing models. For each factor pricing model, the table reports the estimated bs, the J_T statistic, and the p-value. The J_T statistics are calculated from equation (7.14). Under the null hypothesis that the particular physical investment factor pricing model holds, all of the orthogonality conditions will be satisfied and the J_T statistic should be equal to zero. The first two rows of table 7.1 report the GMM results for the first two factor pricing models, in which one and two aggregated physical investment returns are used to construct the stochastic discount factor m. The J_T statistic is 20.76 for the first model and 21.92 for the second model. Both statistics are significantly higher than the 5% critical value, leading to the rejection of the null hypotheses. Hence, the factor pricing models using highly aggregated physical investment returns as factors fail to capture the cross-industry variations in expected equity returns. The last two rows of table 7.1 report the GMM results for the last two factor pricing models, in which less aggregated physical investment returns are used to construct the stochastic discount factor m. The J_T statistics are 11.56 and 2.88, respectively. Since both statistics are substantially lower than the 5% critical values, one can not reject the null hypothesis that the corresponding

Table 7.1 Testing Statistics of the Linear Factor Pricing Models

	b_0	b_1	b_2	b_3	b_4	b_5	b_6	J_T	P
Model 1	0.94	-0.03						20.76	0.02
	(0.57)	(0.52)							
Model 2	-1.22	3.80	-1.77					21.92	0.01
	(2.76)	(3.91)	(1.49)						
Model 3	-0.80	-0.18	-1.56	3.35				11.56	0.17
	(4.43)	(2.11)	(6.06)	(3.50)					
Model 4	13.98	1.72	1.76	0.16	-3.69	6.61	-18.64	2.88	0.72
	(14.13)	(3.20)	(10.02)	(7.02)	(10.61)	(7.08)	(15.62)		

physical investment factor pricing model holds. Therefore, the factor pricing models using disaggregated physical investment returns as factors perform well in explaining the cross-industry variations in expected equity returns. This further confirms the result reported in chapter 6 that the industry physical investment returns contain sufficient information to correctly price the corresponding industry equity portfolio returns.

The above results indicate that the empirical performance of the linear factor pricing model is sensitive to different aggregations of physical investment returns. This highlights the joint hypothesis test problem embedded in the traditional approach adopted in the empirical PCAPM literature. Although using one or two highly aggregated physical investment returns as factors provides a parsimonious factor pricing model, there is no reason to believe that all of the intertemporal physical investment opportunities in the economy can be well captured by one or two aggregate production technologies. Assuming that a couple of physical investment return factors will suffice is not a prediction of the PCAPM theory, but an additional modeling assumption. Hence, once the factor pricing model is rejected by the data, one is not clear whether the rejection comes from the violation of the spanning assumption, or from the inappropriate aggregation of production technologies. By incorporating relatively disaggregated physical investment returns into the model, one can at least alleviate the above joint hypothesis problem

and draw more robust conclusion about the validity of the PCAPM.

7.3 Relationship between the State Price Density Approach and the Linear Factor Pricing Approach

In this section I discuss how the state price density approach proposed in chapter 4 is related to the linear factor pricing approach adopted in this chapter. Both approaches are used to examine the pricing relationship between the physical investment returns and the equity returns. Specifically, one focuses on whether physical investment returns contain sufficient information that can be used to correctly price equity returns.

The traditional linear factor pricing approach examines the performance of a linear physical investment factor pricing model in explaining the cross-sectional variations in expected equity returns. On the other hand, the state price density approach investigates the validity of the spanning assumption, which states that the payoff space of physical investment spans the payoff space of financial investment. The spanning assumption and the physical investment factor pricing model are closely related to each other. If the spanning assumption holds, then the law of one price implies that there exists a stochastic discount factor that can be written as a linear combination of the physical investment returns. Hence, the validity of the physical investment factor pricing model is an immediate implication of the spanning assumption and the law of one price. Moreover, both approaches construct the estimation and testing procedures based on the no-arbitrage constraints. While the linear factor pricing approach estimates and tests a parametric specification of the stochastic discount factor, the state price density approach uses nonparametric techniques to compare the SPDs recovered from physical investment returns and equity returns. Since the stochastic discount factor and the SPD have a one-to-one correspondence in the absence of arbitrage, the two methods can be viewed as dual approaches to examining the pricing relationship between the physical

capital investment and the financial investment.

One advantage the state price density approach has over the linear factor pricing approach lies in the fact that the stochastic discount factor m in the physical investment factor pricing model may be negative. I compute the estimated stochastic discount factor in equation (7.20), and find that m takes negative values in 11 out of 49 years. Even if one puts nonnegativity constraint for m in the GMM estimation to ensure that m takes on positive values in the sample, there is no guarantee that out-of-sample m still remains positive. Negative m implies that there exist arbitrage opportunities in the economy. This is not a desirable property for any asset pricing model. It seems puzzling that one starts from no-arbitrage constraints for both the physical capital investment and the financial investment, but ends up with a factor pricing model allowing for arbitrage opportunities.

The conflicting results come from the fact that the law of one price is less restrictive than the absence of arbitrage. Recall from section 4.1 that the existence of the linear factor pricing model (4.3) is guaranteed by the law of one price. However, the law of one price is a weaker constraint than the absence of arbitrage. The law of one price only requires that two assets with identical future payoff structures should have identical prices. It does not cover cases in which one asset dominates another but may do so by different amounts in different states. Hence, the law of one price alone does not preclude possible arbitrage opportunities in the economy. This is equivalent to say that the stochastic discount factor in (4.3) may be negative in some states. Therefore, although the spanning assumption and the law of one price guarantee the existence of a stochastic discount factor of the form (4.4), such an m is not likely to be the one prevailing in an economy without any arbitrage opportunity.

The state price density approach, on the other hand, focuses on the key assumption leading to the existence of a stochastic discount factor of the form (4.4), namely the spanning assumption. Instead of assuming a parametric form for m, the proposed

nonparametric procedure recovers and compares the SPDs for the physical investment market and the financial market. The positivity and additivity constraints are explicitly incorporated into the optimization problem to ensure that the estimated SPD is the legitimate Radon-Nikodym derivative between the risk-neutral (or equivalent martingale) measure and the actual probability measure. From equation (4.15) one can see that the estimated SPD follows a strictly positive and quite flexible generalized exponential density. Therefore, the state price density approach is inherently consistent with the no-arbitrage constraints. Moreover, since the proposed nonparametric procedure does not impose much restriction on the structure of the stochastic discount factor or the risk-neutral measure, the estimation and testing results are more robust than the linear factor pricing approach.

As concluded in section 7.2.2, over-aggregation of production technologies (or physical investment returns) may lower the explanatory power of the factor pricing model. Hence, it may be desirable in many applications to include disaggregated physical investment returns as factors. The GMM estimation only allows limited flexibility in expanding the set of factors. Since the number of unknown parameters in the physical investment factor pricing model increases linearly with the number of factors included, one needs at least as many orthogonality conditions to perform parameter estimation. To conduct the J_T test, the number of orthogonality conditions must exceed the number of factors. In many cases, it may well be that the number of securities is less than the desirable number of factors. One way to expand the set of orthogonality conditions is to include more instruments. However, the selection of instruments has always been problematic and bedeviled the applications of GMM. Moreover, simulation results have shown that the small-sample properties of GMM estimates and test statistics deteriorate with the number of instruments.

Partly due to the above reasons, most empirical applications adopting the linear factor pricing approach use only one or two highly aggregated production technologies

to construct the stochastic discount factor, and thus suffer from the potential joint hypothesis test problem. On the other hand, the state price density approach proposed in chapter 4 does not put any restriction on the number of physical investment returns or the number of equity returns involved in the study. The entropic technique allows me to extract pricing information separately from the physical investment market and the financial market. Such separation provides much more flexibility in incorporating disaggregated physical investment returns than the GMM estimation. The entropic procedure can be easily applied to cases when the number of physical investment returns (factors) exceeds the number of no-arbitrage constraints (moment conditions) for equity returns.

8. CONCLUSION AND FUTURE RESEARCH

8.1 Summary of Results

In this paper I examine the pricing relationship between physical investment returns and equity returns using industry-level data. One commonly used approach to empirically testing the implications of the PCAPM is to study the validity of a linear factor pricing model, in which highly aggregated physical investment returns are the only factors used for pricing equity returns. However, the physical investment factor pricing model is not always consistent with the spirit of no-arbitrage since the stochastic discount factor may take negative values. Moreover, the traditional approach suffers from a joint hypothesis test problem because the performance of the factor pricing model depends both on the validity of the spanning assumption and on the spanning ability of the selected production technologies.

Based on entropic principles and no-arbitrage constraints, I propose a nonparametric test to study whether the payoff space of physical investment spans the payoff space of financial securities. The proposed test recovers and compares the SPD for both the physical investment market and the stock market. The spanning assumption can not be rejected if the two SPDs are not significantly different from each other. In this case, there is supporting evidence that industry physical investment returns contain sufficient information to correctly price the corresponding equity portfolio returns. Otherwise, the spanning assumption and the physical investment factor pricing model have to be rejected.

I use industry-level data to empirically test the validity of the spanning assumption. Time series of annual physical investment returns and equity returns are constructed for each of the following six industries: mining, construction, manufacturing, transportation, communication, and public utilities. The empirical results show that the SPD recovered from the physical investment returns is able to correctly (in a statistical sense) price the corresponding equity returns. This provides supportive evidence that the payoff space of physical investment spans the payoff space of financial securities. Robustness check shows that the above results are not sensitive to a wide range of parameter values and different function forms for adjustment costs.

I also apply the same data to test the validity of several physical investment factor pricing models. The factor pricing models using disaggregated industry physical investment returns as factors perform well in explaining the cross-industry variations in expected equity returns. This confirms the previous result that physical investment returns contain sufficient information that can be used to correctly price equity returns. However, further study reveals that the physical investment factor pricing model is not always consistent with no-arbitrage condition because the realized stochastic discount factor takes on negative values in some years.

The empirical findings in this study highlight the fact that physical capital investment conveys important information on financial asset pricing. The empirical failure of the traditional CAPM and the CCAPM may be due to the fact that neither the market return nor consumption decisions are able to capture some important intertemporal physical investment opportunities in the economy. To explain both the time-series and the cross-sectional variations in expected equity returns, more works need to be done to explicitly model the impacts of key production characteristics (e.g., the adjustment cost) on asset prices. Such models will generate much richer testable implications than models focusing exclusively on the financial sector of the economy.

The work presented in this study as well as Cochrane (1996) and Kasa (1997) is

actually closely related to the empirical q-theory literature. Tobin's q is defined as the price of existing capital relative to new capital. The relationship between Tobin's q and physical investment expenditure has been a central topic of empirical investment literature. The q-theory of investment (Tobin (1969) and Tobin and Brainard (1977)) states that the firm's demand for new capital investment, as measured by its physical investment expenditures, should be positively correlated with the market value of existing capital stock relative to its replacement cost (the average q). In other words, the q-theory predicts that the firm should increase its physical capital investment as long as the market valuation of physical capital exceeds the investment cost. Hence, the time-series and cross-sectional variations in physical capital investment can be explained by the changes in the market value of firm's capital stock. Hayashi and Inoue (1991) construct a tax-adjusted measure of q and test the q-theory of investment using panel data on firms from Japan. The empirical results show that q is a significant determinant of physical capital investment. Blundell, Bond, Devereux, and Schiantarelli (1992) examine the q-theory using panel data on firms from the United Kingdom, and find that the coefficient on q is significant but small.

Compared with the empirical q-theory literature, the approach adopted by the empirical studies on PCAPM actually reverses the logic behind the q-theory of capital investment. Instead of using stock market information to explain the physical capital investment, the empirical PCAPM literature uses the change of firms' physical capital investment decisions to explain the time-series and cross-sectional variations in equity returns. In the PCAPM, the fundamental source of uncertainty in both the physical investment market and the stock market is the business cycle caused by macroeconomic risks. Since the intertemporal nature of production is a central determinant of the course of real business fluctuation, the variations in equity returns should be determined by the effects of macroeconomic risks on physical investment decisions. Consequently, physical capital investment should contain crucial information necessary for correctly pricing fi-

nancial securities. The empirical results documented in this study provide supporting evidence for the above argument since the state price density recovered from physical investment returns is able to price the corresponding equity returns. Clearly, the empirical q-theory literature and the empirical PCAPM literature are just two sides of the same coin, and are therefore consistent with each other.

8.2 Future Research

In this paper I propose a nonparametric procedure to study the pricing relationship between the physical investment returns and the equity returns. In particular, I examine whether the SPD recovered from the physical investment returns can be used to correctly price the equity returns. One may notice that I have exclusively focused on recovering and comparing unconditional SPDs (or risk-neutral probability measures). However, if one worries about things like GARCH effects, then it may be the case that conditional risk-neutral probability measures are of more interest. The conditional probability measure refers to the distribution of a return (either the physical investment return or the equity return) at time T conditional on all the information available at time T. The information set at time T includes at least all of the return realizations in the past. Hence, one direction of future research is to find ways to recover and compare the conditional risk-neutral measures for the physical investment returns and the equity returns.

Unfortunately, the method for recovering the conditional risk-neutral measure from a time series of returns is not a trivial extension of the unconditional method. Somewhat surprisingly, the estimation of conditional risk-neutral density has not been explored much in the empirical asset pricing literature. There are some works in the option pricing literature estimating the conditional SPDs for the underlying stock prices. However, the technique used relies on one convenient relation between option prices and SPDs, which suggests that the second derivative of the call-pricing function with respect to the strike

price must equal to the SPD. This property was first discovered by Ross (1976), Banz and Miller (1978), and Breeden and Litzenberger (1978). Ait-Sahalia and Lo (1998) construct a nonparametric call-pricing formula, and applies the above property to derive an estimate for the conditional SPD.

Clearly, the above approach can not be generalized to other cases when the stated relationship between asset prices and conditional SPD does not hold. More generally, if one does not want to impose parametric assumptions on the return generating process, the problem to be solved can be stated as the problem of efficient non-parametric estimation of the conditional SPD (or risk-neutral measure) subject to some conditional moment conditions (e.g., no-arbitrage constraints). Let $R(t)$ be a $N \times 1$ random vector of asset returns for $t = 0, 1, \ldots, T$ and $f(\cdot)$ be the density function for the risk-neutral measure. The following three-step procedure provides one way to estimate the conditional risk-neutral measure.

1. Construct a non-parametric estimate of the joint risk-neutral density $f(R(T), R(T - 1), \ldots, R(0))$ for all the return vectors subject to the conditional no-arbitrage constraints.

2. Estimate the marginal density $f(R(T - 1), \ldots, R(0))$ for all the return vectors dated in the past based on the joint density estimate from step 1.

3. Estimate the conditional risk-neutral density by utilizing the definition of conditional density:

$$f(R(T)|R(T - 1), \ldots, R(0)) = \frac{f(R(T), R(T - 1), \ldots, R(0))}{f(R(T - 1), \ldots, R(0))}. \qquad (8.1)$$

Implementing the above procedure is obviously not a trivial task. Especially for step 1, one needs to figure out how to efficiently incorporate all of the information in the conditional constraints into the estimation of the joint risk-neutral density.

For future research, I would like to see how the entropic approach proposed in chapter 4 can be modified to recover and compare the conditional SPDs for the physical investment returns and the equity returns. Examining the spanning assumption from a conditional perspective will contribute to both the empirical PCAPM literature and the conditional density estimation in the empirical asset pricing literature.

As another extension to the current research, I would like to examine how differences in adjustment cost are related to the cross-industry variations in equity returns. As pointed out in chapter 1, the existence of adjustment cost may play an important role in determining the price persistence, the volatility, and the risk premia of risky assets. Since the cost of adjusting capital stock varies significantly across industries, a cross-industry study will shed light on the direction and magnitude of the pricing impact of adjustment cost. Such study may also provide a list of stylized facts that should be captured by any model focusing on the impacts of key production characteristics on asset prices.

APPENDIX A.   ENTROPIC PRINCIPLES

This appendix provides a brief introduction to entropic principles and derives the solution to the maximum entropy problem and the cross-entropy minimization problem. For a rigorous and complete description of entropy concept and its applications in economics and finance, please refer to Golan, Judge and Miller (1996).

The entropy measure originates from physics. It was first proposed in the 1870s to measure the information in a distribution that defines the thermodynamic state of a physical system. In an information theoretic context, it is used to measure uncertainty or missing information. Shannon (1948) first proposes to use the entropy measure to gauge the uncertainty embedded in a noisy message. Based on Shannon's entropy metric, Jaynes (1957) develops a maximum entropy principle that forms a basis for estimation and inference of ill-posed, pure inverse problem. In the presence of prior knowledge, Good (1963) proposes to use the minimum cross-entropy principle to ensure that the estimation and inference are consistent with both the information in the data and the prior belief. Both the maximum entropy principle and the cross-entropy minimization principle have been used as effective information processing rules when the observed sample data are limited and aggregated, and when the underlying sampling model is incomplete or incorrectly specified. For example, in this study I examine whether the state price density (or risk-neutral measure) recovered from the physical investment returns can be used to correctly price the equity returns. Without any modeling assumption, the only information we have is the return series. Given the observed return series, the state price densities satisfying the no-arbitrage constraints are in general not unique. To

identify the state price density that is most consistent with the incomplete information we have, we need some sort of information processing rule to make an optimal selection from the feasible set of state price densities. Hobson (1971) shows that, under some axiomatic conditions, entropy criterion is the most efficient (in terms of information processing) rule that we should adopt. In the following sections, I will illustrate the maximum entropy principle and the minimum cross-entropy principle in the context of recovering the unknown probability distribution from the incomplete information (data) at hand. I will first describe each principle in discrete case, and then extend to the continuous case.

Shannon's Entropy

Suppose that there are S possible outcomes for a future event with a discrete probability distribution $\mathbf{p} \equiv (p(1), p(2), \ldots, p(S))$. To measure the uncertainty of the above random event, Shannon (1948) uses an axiomatic method to define the entropy of the probability distribution $\mathbf{p}$ as

$$H(\mathbf{p}) \equiv -\sum_{s=1}^{S} p(s) \log p(s). \tag{A.1}$$

Here $-\log(\cdot)$ can be interpreted as an information score measuring the information gathered from observing a particular outcome. The negative log functional form implies that the information score of a particular outcome is inversely proportional to its probability. By averaging the information scores over all possible outcomes, $H(\mathbf{p})$ gives us the expected information gained from the occurrence of a future event. Note that $H(\mathbf{p})$ reaches its maximum when the possible outcomes are uniformly distributed. In this case, one is completely uncertain about which outcome will occur.

Shannon's entropy is also closely related to the concept of maximum likelihood estimation. Suppose that nature carries out K trials with S possible outcomes for each trial. Let $k_1, k_2, \ldots, k_S$ be the number of times each outcome occurs. Note that $\sum_{s=1}^{S} k_s = K$.

Further, let W be the total number of ways a particular $(k_1, k_2, \ldots, k_S)$ can be realized in K trials, i.e.,

$$W \equiv \frac{K!}{k_1! k_2! \ldots k_S!}. \tag{A.2}$$

Golan, Judge and Miller (1996) show that

$$K^{-1} \log W \approx H(\mathbf{p}) \tag{A.3}$$

if K is large enough. Therefore, maximizing Shannon's entropy is approximately equivalent to choosing a probability measure $(p(1), p(2), \ldots, p(S))$ that can be realized in the greatest number of ways. This is consistent with the spirit of maximum likelihood estimation.

Maximum Entropy Principle

In many cases, we need to recover a probability distribution from a given set of moment constraints. In general the feasible probability distributions satisfying the moment constraints are not unique. The problem of selecting a particular probability distribution from the feasible set is said to be ill-posed or undetermined. In discrete state case, the ill-posed problem often takes the form that the number of states of the world exceeds the number of moment constraints.

As before, we assume that there are finite number of states of the world, denoted by s for $s = 1, 2, \ldots, S$. Suppose that there are a total of N $(N < S)$ moment constraints for the random variable x, and that these moment constraints are the only information available:

$$\sum_{s=1}^{S} p(s) f_i(x(s)) = y_i, \qquad i = 1, 2, \ldots, N, \tag{A.4}$$

where $f_i(\cdot)$ is a function of random variable x. The problem of recovering $\mathbf{p}$ is ill-posed because the number of states of the world S exceeds the number of constraints N.

To select a particular probability distribution which is the best estimate of the unknown p, Jaynes (1957a,b) proposes to solve the following maximization problem:

$$\max_{\mathbf{p}} H(\mathbf{p}) \equiv -\sum_{s=1}^{S} p(s) \log p(s) \tag{A.5}$$

subject to

$$\sum_{s=1}^{S} p(s) f_i(x(s)) = y_i, \qquad i = 1, 2, \ldots, N, \tag{A.6}$$

$$\sum_{s=1}^{S} p(s) = 1, \tag{A.7}$$

$$p(s) > 0, \qquad s = 1, 2, \ldots, S. \tag{A.8}$$

To recover the probability distribution p, one can form the Lagrangian function

$$L = -\sum_{s=1}^{S} p(s) \log p(s) + \sum_{i=1}^{N} \lambda_i [y_i - \sum_{s=1}^{S} p(s) f_i(x(s))] + \lambda_0 (1 - \sum_{s=1}^{S} p(s)) \tag{A.9}$$

with first-order conditions

$$\frac{\partial L}{\partial p(s)} = -\log p(s) - 1 - \sum_{i=1}^{N} \lambda_i f_i(x(s)) - \lambda_0 = 0, \quad s = 1, 2, \ldots, S, \tag{A.10}$$

$$\frac{\partial L}{\partial \lambda_i} = y_i - \sum_{s=1}^{S} p(s) f_i(x(s)) = 0, \qquad i = 1, 2, \ldots, N, \tag{A.11}$$

$$\frac{\partial L}{\partial \lambda_0} = 1 - \sum_{s=1}^{S} p(s) = 0, \tag{A.12}$$

where $\boldsymbol{\lambda} \equiv (\lambda_0, \lambda_1, \ldots, \lambda_N)$ are the Lagrange multipliers. Manipulating terms in equations (A.10), (A.11) and (A.12) yields

$$\hat{p}(s) = \exp\left[-\sum_{i=1}^{N} \hat{\lambda}_i f_i(x(s)) - 1 - \hat{\lambda}_0\right], \qquad s = 1, 2, \ldots, S, \tag{A.13}$$

$$y_i = \sum_{s=1}^{S} \exp\left[-\sum_{i=1}^{N} \hat{\lambda}_i f_i(x(s)) - 1 - \hat{\lambda}_0\right] f_i(x(s)), \quad i = 1, \ldots, N, \tag{A.14}$$

$$1 = \sum_{s=1}^{S} \exp\left[-\sum_{i=1}^{N} \hat{\lambda}_i f_i(x(s)) - 1 - \hat{\lambda}_0\right]. \tag{A.15}$$

Equation (A.15) implies that

$$\exp(1 + \hat{\lambda}_0) = \sum_{s=1}^{S} \exp\left[-\sum_{i=1}^{N} \hat{\lambda}_i f_i(x(s))\right]. \tag{A.16}$$

Substituting equation (A.16) into equation (A.13) gives

$$\hat{p}(s) = \frac{\exp\left[-\sum_{i=1}^{N}\hat{\lambda}_i f_i(x(s))\right]}{\Omega(\hat{\lambda})},\tag{A.17}$$

where

$$\Omega(\hat{\lambda}) \equiv \sum_{s=1}^{S}\exp\left[-\sum_{i=1}^{N}\hat{\lambda}_i f_i(x(s))\right].\tag{A.18}$$

The Lagrange multipliers are determined by the following equations:

$$y_i = \left(\frac{\partial}{\partial \lambda_i}\right)\log\Omega, \qquad i = 1, 2, \ldots, N.\tag{A.19}$$

The optimum value of the entropy measure H can be derived by substituting equation (A.17) into equation (A.5):

$$H(\hat{\lambda}) = \log\Omega(\hat{\lambda}) + \sum_{i=1}^{N}\hat{\lambda}_i y_i.\tag{A.20}$$

Golan, Judge and Miller (1996) show that the above maximization problem has a unique solution because the Hessian matrix is negative definite. Further, the solution $\hat{p}$ satisfies both the additivity and the positivity constraints. Note that $\hat{p}$ depends on the Lagrange multiplier $\hat{\lambda}$. Under the current problem setup, there is no closed-form solution for $\hat{\lambda}$ and the solution must be obtained numerically.

The above maximum entropy formulation allows us to select a probability distribution that only describes what we know (the information incorporated in the moment constraints). The solution $\hat{p}$ is the best estimate possible in the sense that it can be realized in the greatest number of ways consistent with all the information we have.

Suppose now that x is a continuous random variable with probability density function $p(x)$. As a straightforward extension of the discrete case maximum entropy principle, the continuous formalism can be stated as

$$\max_{p} H(p) \equiv -\int p(x)\log p(x)dx\tag{A.21}$$

subject to

$$\int p(x)f_i(x)dx = y_i, \qquad i = 1, 2, \ldots, N, \tag{A.22}$$

$$\int p(x)dx = 1, \tag{A.23}$$

$$p(x) \geq 0. \tag{A.24}$$

To recover the probability density function $p(x)$, one can form the Lagrangian function

$$
\begin{aligned}
L &= -\int p(x)\log p(x)dx + \sum_{i=1}^{N} \lambda_i[y_i - \int p(x)f_i(x)dx] + \lambda_0(1 - \int p(x)dx) \\
&= \sum_{i=1}^{N} \lambda_i y_i + \lambda_0 + \int[-p(x)\log p(x) - p(x)\sum_{i=1}^{N}\lambda_i f_i(x) - \lambda_0 p(x)]dx. \tag{A.25}
\end{aligned}
$$

By using the calculus of variations, the first-order condition with respect to $p(x)$ is given by

$$-\log p(x) - 1 - \sum_{i=1}^{N}\lambda_i f_i(x) - \lambda_0 = 0. \tag{A.26}$$

Similar to the discrete case, the first-order conditions with respect to the Lagrangian multipliers are given by

$$y_i - \int p(x)f_i(x)dx = 0, \qquad i = 1, 2, \ldots, N, \tag{A.27}$$

$$1 - \int p(x)dx = 0. \tag{A.28}$$

Equation (A.26) implies that

$$\hat{p}(x) = \frac{\exp(-\sum_{i=1}^{N}\hat{\lambda}_i f_i(x))}{\exp(1 + \hat{\lambda}_0)}. \tag{A.29}$$

Substituting equation (A.29) into equation (A.28) gives

$$\exp(1 + \hat{\lambda}_0) = \int \exp(-\sum_{i=1}^{N}\hat{\lambda}_i f_i(x))dx. \tag{A.30}$$

Combining equations (A.29) and (A.30) gives us the optimal solution for $p(x)$:

$$\hat{p}(x) = \frac{\exp(-\sum_{i=1}^{N}\hat{\lambda}_i f_i(x))}{\Omega(\hat{\lambda})}, \tag{A.31}$$

where

$$\Omega(\hat{\boldsymbol{\lambda}}) \equiv \int \exp(-\sum_{i}^{N} \lambda_i f_i(x))dx. \tag{A.32}$$

As in the discrete state case, the Lagrange multiplier $\hat{\boldsymbol{\lambda}}$ does not have a closed-form solution and must be obtained numerically.

Minimum Cross-Entropy Principle

Besides the data, we sometimes have prior beliefs or non-sample information on the unknown probability distribution $\mathbf{p}$. Suppose that the non-sample information takes the form of a probability vector $\mathbf{q} \equiv (q(1), q(2), \ldots, q(S))$ in the discrete case. The question then becomes how to choose the best estimate of the unknown probability measure $\mathbf{p}$ based on the moment constraints (A.6) and the prior information $\mathbf{q}$. Unlike the maximum Shannon-entropy framework, Good (1963) proposes to minimize the cross-entropy between the probability measures consistent with the data information and the prior information $\mathbf{q}$.

The cross-entropy $I(\mathbf{p}, \mathbf{q})$ between the two probability measures $\mathbf{p}$ and $\mathbf{q}$ is defined as

$$I(\mathbf{p}, \mathbf{q}) \equiv \sum_{s=1}^{S} p(s) \log \left(\frac{p(s)}{q(s)} \right). \tag{A.33}$$

The concept was first developed by Kullback and Leibler (Kullback (1959)), and is also known as the Kullback-Leibler Information Criterion (KLIC). Clearly, $I(\mathbf{p}, \mathbf{q}) = 0$ if $\mathbf{p}$ and $\mathbf{q}$ are identical. Otherwise, it can be shown that $I(\mathbf{p}, \mathbf{q}) > 0$. In the special case that $\mathbf{q}$ is a uniform distribution, $I(\mathbf{p}, \mathbf{q}) = \log(S) - H(\mathbf{p})$.

Following Good (1963), the unknown probability distribution $\mathbf{p}$ can be recovered by solving the following minimization problem:

$$\min_{\mathbf{p}} I(\mathbf{p}, \mathbf{q}) \equiv \sum_{s=1}^{S} p(s) \log \left(\frac{p(s)}{q(s)} \right) \tag{A.34}$$

subject to equations (A.6), (A.7) and (A.8).

The Lagrangian function can be written as

$$L = \sum_{s=1}^{S} p(s) \log\left(\frac{p(s)}{q(s)}\right) + \sum_{i=1}^{N} \lambda_i[y_i - \sum_{s=1}^{S} p(s) f_i(x(s))] + \lambda_0(1 - \sum_{s=1}^{S} p(s)) \qquad (A.35)$$

with the first-order conditions $\partial L/\partial(\cdot) = 0$. Carrying through the same steps as with the maximum entropy problem, we can solve $\mathbf{p}$ from the first-order conditions as

$$\hat{p}(s) = \frac{q(s) \exp\left[\sum_{i=1}^{N} \hat{\lambda}_i f_i(x(s))\right]}{\Omega(\hat{\boldsymbol{\lambda}})}, \qquad s = 1, 2, \ldots, S, \qquad (A.36)$$

where

$$\Omega(\hat{\boldsymbol{\lambda}}) \equiv \sum_{s=1}^{S} q(s) \exp\left[\sum_{i=1}^{N} \hat{\lambda}_i f_i(x(s))\right]. \qquad (A.37)$$

Again, there is no closed-form solution for the Lagrange multiplier $\hat{\boldsymbol{\lambda}}$ and the solution has to be found numerically.

The minimum cross-entropy framework guarantees that, among all the probability distributions satisfying the moment constraints, the optimal solution $\hat{\mathbf{p}}$ is the one closest to the prior information $\mathbf{q}$. The optimal solution $\hat{\mathbf{p}}$ is selected in such a way that no information other than the moment constraints are incorporated in the process of updating the prior $\mathbf{q}$.

The relationship between the maximum Shannon-entropy and the minimum cross-entropy is as follows. The maximum Shannon-entropy is nested in the minimum cross-entropy framework. To see this, recall that $I(\mathbf{p}, \mathbf{q}) = \log(S) - H(\mathbf{p})$ if $\mathbf{q}$ is a discrete uniform distribution. In this case, minimizing the cross-entropy $I(\mathbf{p}, \mathbf{q})$ is equivalent to maximizing the Shannon-entropy $H(\mathbf{p})$.

Suppose now that we have a continuous random variables x with probability density function $p(x)$. Define $q(x)$ as the prior belief of the probability density function. The continuous version of minimum cross-entropy principle can then be stated as

$$\min_{p} \int p(x) \log(\frac{p(x)}{q(x)}) dx = \int p(x) \log p(x) dx - \int p(x) \log q(x) dx \qquad (A.38)$$

subject to

$$\int p(x)f_i(x)dx = y_i, \qquad i = 1, 2, \dots, N, \tag{A.39}$$

$$\int p(x)dx = 1, \tag{A.40}$$

$$p(x) \geq 0. \tag{A.41}$$

The Lagrangian function can be written as

$$
\begin{aligned}
L &= \int p(x)\log p(x)dx - \int p(x)\log q(x)dx + \sum_{i=1}^{N}\lambda_i[y_i - \int p(x)f_i(x)dx] \\
&\quad +\lambda_0(1 - \int p(x)dx) \\
&= \sum_{i=1}^{N}\lambda_i y_i + \lambda_0 \\
&\quad + \int [p(x)\log p(x) - p(x)\log q(x) - p(x)\sum_{i=1}^{N}\lambda_i f_i(x) - \lambda_0 p(x)]dx. \tag{A.42}
\end{aligned}
$$

Using the calculus of variations, the first-order condition with respect to $p(x)$ can be expressed as

$$\log p(x) + 1 - \log q(x) - \sum_{i=1}^{N}\lambda_i f_i(x) - \lambda_0 = 0. \tag{A.43}$$

Manipulating terms gives

$$\hat{p}(x) = q(x)\exp[\sum_{i=1}^{N}\hat{\lambda}_i f_i(x) + \hat{\lambda}_0 - 1]. \tag{A.44}$$

The first-order conditions with respect to the Lagrange multipliers are given by

$$y_i - \int p(x)f_i(x)dx = 0, \qquad i = 1, 2, \dots, N, \tag{A.45}$$

$$1 - \int p(x)dx = 0. \tag{A.46}$$

Substituting equation (A.44) into (A.46) gives

$$\exp(\hat{\lambda}_0 - 1) = \frac{1}{\int q(x)\exp[\sum_{i=1}^{N}\hat{\lambda}_i f_i(x)]dx} \tag{A.47}$$

Combining equations (A.44) and (A.47) gives the solution for $p(x)$ as

$$\hat{p}(x) = \frac{q(x)\exp[\sum_{i=1}^{N}\hat{\lambda}_i f_i(x)]}{\Omega(\hat{\lambda})}, \tag{A.48}$$

where

$$\Omega(\hat{\lambda}) \equiv \int q(x) \exp[\sum_{i=1}^{N} \hat{\lambda}_i f_i(x)] dx. \qquad \text{(A.49)}$$

Denote P and Q as the cumulative probability distribution function corresponding to $p(x)$ and $q(x)$, respectively. Then the solution (A.48) can be rewritten as

$$\frac{d\hat{P}}{dQ} = \frac{\exp[\sum_{i=1}^{N} \hat{\lambda}_i f_i(x)]}{\Omega(\hat{\lambda})}, \qquad \text{(A.50)}$$

where

$$\Omega(\hat{\lambda}) \equiv E_Q\{\exp[\sum_{i=1}^{N} \hat{\lambda}_i f_i(x)]\}. \qquad \text{(A.51)}$$

Here E_Q denotes the expectation taken with respect to the probability distribution Q. The Lagrange multiplier $\hat{\lambda}$ can be found by solving the following convex minimization problem:

$$\hat{\lambda} \equiv \arg \min_{\lambda} \Omega(\lambda). \qquad \text{(A.52)}$$

APPENDIX B. NO-ARBITRAGE AND ASSET PRICING

In this appendix, I briefly review asset pricing representations under the absence of arbitrage. For illustration purpose, I assume that the states of the world are finite and discrete. All the major results can be extended to the case when the states of the world are continuous.

The asset pricing theories in modern finance literature are based on the assumption that no arbitrage opportunities are available in equilibrium. The following **Fundamental Theorem of Asset Pricing** (Dybvig and Ross (1992)) states the implication of no-arbitrage on asset pricing.

Theorem 1 *The following are equivalent:*

- *Absence of arbitrage*

- *Existence of a positive linear pricing rule*

- *Existence of an optimal demand for some agent who prefers more to less*

Assume that there are finite number of states of the world, denoted by s for $s = 1, 2, \ldots, S$, and that there are finite number of risky assets, denoted by i for $i = 1, 2, \ldots, N$. According to the above theorem, the absence of arbitrage implies that

$$\sum_{s=1}^{S} \psi(s) R_i(s) = 1, \qquad i = 1, 2, \ldots, N, \qquad (\text{B.1})$$

where $\psi(s)$ is the positive state price that correctly prices all assets in state s, and $R_i(s)$ is the gross rate of return for asset i if state s occurs. The $\psi(s)$ is the positive linear

pricing rule referred to in the Fundamental Theorem of Asset Pricing. It is the current price of an Arrow security which promises to pay one dollar in state s and zero dollar in all other states.

Several other alternative representations of the basic linear pricing rule are also available. Among these, the risk-neutral (or martingale) representation and the stochastic discount factor representation are most frequently used. The choice of a particular representation depends on the specific context of the problem under investigation. The risk-neutral representation (Cox and Ross (1976), Harrison and Kreps (1979)) is particularly useful for optimization problems without reference to individual preferences , while the stochastic discount factor representation (Cox and Leland (1982), Dybvig (1980, 1985)) is most useful when dealing with choice problems .

To derive the risk-neutral representation, let us divide both sides of equation (B.1) by $\sum_{s=1}^{S} \psi(s)$:

$$\sum_{s=1}^{S} \frac{\psi(s)}{\sum_{s=1}^{S} \psi(s)} R_i(s) = \frac{1}{\sum_{s=1}^{S} \psi(s)}, \qquad i = 1, 2, \ldots, N. \tag{B.2}$$

Let $\pi^*(s) \equiv \frac{\psi(s)}{\sum_{s=1}^{S} \psi(s)}$. Then equation (B.2) can be written as

$$\sum_{s=1}^{S} \pi^*(s) R_i(s) = \frac{1}{\sum_{s=1}^{S} \psi(s)}, \qquad i = 1, 2, \ldots, N. \tag{B.3}$$

It is easy to see that $\pi^*(s) > 0$ for all s and that $\sum_{s=1}^{S} \pi^*(s) = 1$. Therefore, π^* can be interpreted as an artificial probability measure. Furthermore, it can be shown that $\sum_{s=1}^{S} \psi(s) = r^{-1}$ if there exists a gross riskless rate r. The above arguments allow us to write equation (B.3) in the following equivalent way:

$$E_{\pi^*} \left[\frac{1}{r} R_i \right] = 1, \qquad i = 1, 2, \ldots, N, \tag{B.4}$$

where E_{π^*} denotes the expectation under the artificial probability measure π^*. In finance literature, π^* is conventionally referred as the risk-neutral (or martingale) probability measure. It is important to note that π^* is generally different from the true probability measure π over states of the world.

To derive the stochastic discount factor representation, let $m(s) \equiv \psi(s)/\pi(s)$. Here $\pi(s)$ is the true probability that state s will occur. Equation (B.1) can then be written as

$$\sum_{s=1}^{S} \pi(s)m(s)R_i(s) = 1, \qquad i = 1, 2, \ldots, N, \tag{B.5}$$

or, equivalently,

$$E_{\pi}(mR_i) = 1, \qquad i = 1, 2, \ldots, N. \tag{B.6}$$

Here E_{π} denotes the expectation with respect to the true probability measure π over states of the world.

The relationships among the three asset pricing representations are stated in the following **Pricing Rule Representation Theorem** (Dybvig and Ross (1992)).

Theorem 2 *The following are equivalent:*

- *Existence of a positive linear pricing rule*

- *Existence of positive risk-neutral probabilities and an associated riskless rate (the martingale property)*

- *Existence of a positive stochastic discount factor*

Both the risk-neutral representation and the stochastic discount factor representation are used in this study. When testing the spanning assumption on physical investment returns and equity returns, I apply entropic principles to recover and compare the risk-neutral probability measures of the two markets. The risk-neutral representation is used because it fits the entropic framework best among the three pricing rule representations. When studying the validity of the physical investment factor pricing model, I adopt the stochastic discount factor representation since it fits into the GMM framework naturally.

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